Aircraft Loss-of-Control Prevention and Recovery: A Hybrid Control Strategy

A Thesis

Submitted to the Faculty

of

Drexel University

by

Jean - Etienne Temgoua Dongmo

in partial fulfillment of the

requirements for the degree

of

PhD in Mechanical Engineering and Mechanics

June 2010

© Copyright June 2010
Jean - Etienne Temgoua Dongmo.

This work is licensed under the terms of the Creative Commons Attribution-ShareAlike license. The license is available at `http://creativecommons.org/licenses/by-sa/2.0/`.

Acknowledgements

I'd like to thank a few people for their valuable support and contribution towards my research through all these years. First and foremost my academic advisor Dr. Harry G. Kwatny to whom I'm indebted for introducing my to the topic of Aircraft Loss-Of-Control.

I would like to thank Drexel University,the College of Engineering and the Mechanical Engineering & Mechanics for their all times support during my academic years. Special thanks for the MEM faculty and the Control Group for interesting discussion and clarifications on control systems issues. I would like to especially thank Dr. Mun Choi for introducing me into the PhD program and Dr. Christine Belcastro from NASA for her invaluable and timely inputs and also Dr. Gaurav Bajpai and his team from Techno - Sciences.Inc for interesting discussions and its supports. Moreover,I would like to thank Eric Peterson, Mishah Salman for their tremendous support and co - operations. Of course, I'm grateful to my parents for their patience and love. Without them, this work would have never come to existence.

Finally, I wish to thank the following: Ngueguim Suzanne, Nkenfack Fabien for their invaluable support

Dedications

Dedicated to my family and especially Ngueguim Suzanne.

Table of Contents

List of Figures

Abstract

Aircraft Loss-of-Control Prevention and Recovery: A Hybrid Control Strategy

Jean - Etienne Temgoua Dongmo

Advisor: Harry G. Kwatny, PhD

The Complexity of modern commercial and military aircrafts has necessitated better protection and recovery systems. With the tremendous advances in computer technology, control theory and better mathematical models, a number of issues (Prevention, Reconfiguration, Recovery, Operation near critical points, ... etc) moderately addressed in the past have regained interest in the aeronautical industry.

Flight envelope is essential in all flying aerospace vehicles. Typically, flying the vehicle means remaining within the flight envelope at all times. Operation outside the normal flight regime is usually subject to failure of components (Actuators, Engines, Deflection Surfaces) , pilots's mistakes, maneuverability near critical points and environmental conditions(crosswinds...) and in general characterized as ***Loss-Of-Control (LOC)*** because the aircraft no longer responds to pilot's inputs as expected.

For the purpose of this work,***(LOC)*** in aircraft is defined as the departure from the safe set (controlled flight) recognized as the maximum controllable (reachable) set in the initial flight envelope. The LOC can be reached either through failure, unintended maneuvers, evolution near irregular points and disturbances. A coordinated strategy is investigated and designed to ensure that the aircraft can maneuver safely in their constraint domain and can also recover from abnormal regime. The procedure involves the computation of the largest controllable (reachable) set (Safe set) contained in the initial prescribed envelope. The problem is posed as a reachability problem using Hamilton-Jacobi Partial Differential Equation(HJ - PDE) where a cost function is set to be minimized along trajectory departing from the given set. Prevention is then obtained by computing the controller which would allow the flight vehicle to remain in the maximum controlled set in a multi-objective set up. Then the recovery procedure is illustrated with a two - point boundary value problem. Once illustrate, a set of control strategies is designed for recovery purpose ranging from nonlinear smooth regulators with Hamilton Jacobi-Bellman (HJB) formulation to the switching controllers with High Order Sliding Mode Controllers (HOSMC). A coordinated strategy known as a high level supervisor is then implemented using the multi-models concept where models operate in

specified safe regions of the state space.

1. Introduction

The motivation of this work comes from [1] where an overview of the aerodynamic modeling and simulation of commercial aircrafts in upset conditions was laid out experimentally. Based on their report, they conclude that advanced control systems for aerospace vehicles should be smart enough to protect and recover from post upset conditions.

Due to the above fact,modern commercial and military aircraft protection and recovery systems should be designed such that the vehicle is fully protected and can recover without limiting pilot control authority.Such goals can only be achieved if both design and pilot's mistakes are minimized at all operating modes (flight envelope). Such an underlying statement requires understanding of the dynamical model and an investigation of all causes of aircraft accidents throughout the last decade.

1.1 Survey of Aircraft Accident

Aviation safety remains the main concern of National Transportation Safety Board (NTSB), Federation Aviation Authority (FAA), Civil Aviation Authority (CAA) and National Aeronautics and Space Administration (NASA) despite the tremendous reduction in the last decade of the number of airliner accidents. Based on recent reports of the CAA [5], *loss-of-control* remains the only type of accident that has stay steady world-wide statistics with the number of causes varying from non-technical (Pilot's flight handling, bird strike, loading error, contaminated runways, aircraft icing) to technical failure (failures of primary deflection surfaces or critical components) .

According to the forecast [4], air traffic will accelerate in the next 10 years due to an increase of passengers and the trend of replacing wide-body, larger aircraft with smaller, narrow-body planes. The number of fatal accidents will increase if the fatal accident rate does not reduce as fast as air traffic increases. Due to this fact continuous efforts are currently made by the above organizations to lower to their best ability the rate of accidents. As reported by [5] between 1995 and 2004, there were 101 fatal accidents worldwide involving the *loss-of-control (LOC)* among which 85 were associated with primary factors such as maintenance and or icing condition, 59 others were associated with the loss-of-control. Analysis of these accidents shows a number of issues: Loading error, poor risk management by the crew; unsuccessful flight handling near critical points(Stall,Spin,etc...) that are usually connected bifurcation phenomenon, poorly executed go-around and inappropriate used of automation. In the mean time within the same period, 186 fatal accidents occurred due to weather as a primary factor. Among the 186 accidents, 107 were associated with approach and landing while 44 were directly associated to contaminated runway and weather conditions, 22 directly related to wind (wind shear, upsets, turbulence and wind gust) and 14 associated to ice and other factors. Of all these fatal accidents, a close look at their report has directed our attention to some of the critical ones worldwide:

On November 12, 2001, American Airlines Flight 587 (Airbus A300-605R) crashed into a residential area of Belle Harbor, New York, shortly after takeoff from John F. Kennedy International Airport, New York [6] All 260 people aboard the airplane and 5 people on the ground were killed, and the airplane was destroyed by impact forces and a post crash fire. The NTSB determined that the probable cause of this accident was the in-flight separation of the vertical stabilizer.The NTSB also conclude that the separation was a result of the loads beyond ultimate design that were created by the first officer's unnecessary and excessive rudder pedal inputs.

On January 8, 2003, Air Midwest Flight 5481 (Raytheon Beechcraft 1900D) crashed shortly after takeoff from Charlotte-Douglas International Airport, Charlotte, North Carolina. The 2 flight crewmembers and 19 passengers aboard the airplane were killed, 1 person on the ground received minor injuries, and the airplane was destroyed by impact forces and a post crash fire. The National Transportation Safety Board determined that the probable cause of this accident was the airplane's loss of pitch control during takeoff. The loss of pitch control resulted from the incorrect rigging of the elevator control system compounded by the airplane's aft center of gravity.

On February16, 1998, China Airlines Airbus A300-600 crashes at Taipei due to inadequately flown Go-around. This happens because the decision for Go-around has to be taken late in the approach and can result in a high workload situation and a very high load maneuver that can lead to the development of an unusual attitude. Also during that period, the control of maximum thrust is very challenging and the aircraft system is not always helpful which is why this stall occurs 39 seconds after initiating the Go-around process [5].

On January3, 2004, Flash Airlines Boeing 737 craches at Sharm-el-Sheikh for inadequate recovery from extreme attitude due to inability of the crew to steer back to the normal flight envelope after upset conditions. Although special attention is paid to the pilot's training, emphasis should be on the recovery from upset conditions and operation near critical conditions.

Of the last ten years, there have been 20 fatal accidents worldwide involving large public transport airplanes where loading error was a causal factor. Especially one at Guernsey Airport carrying three tons of papers and during the approach it was found that the aircraft was outside the loading limits. The relative proportion of passengers to the cargo in flight is relatively uncorrelated which is one of the leading factor of those fatal accidents.

Along with others, one factor that appears to be critical factor in causing the loss-of-control is the ice contamination of the aircraft before departure. Icing affects performance because it has been found that even a slight roughness reduces the margin between the stick shaker activation and aerodynamic stall. The roughness also creates an out of trim condition such that the aircraft tends to rotate requiring the pilot to exert a 'push force' on the flight controls to maintain the pitch altitude rather than the 'pull force' regularly done in the normal regime [5]. Moreover, the de-icing fluid is reported to have side effects when drying out. These de-icing usually have residues which on contact with water form a gel that may refreeze around control runs producing a stiffness in the controls or even complete jamming of an individual control surface. Although most of the causes cited involve non-technical failures or inappropriate crew response issues, technical issues tend to be the principal source of the loss of control. This is despite the fault tolerant capability designed into modern aircraft. In fact the loss-of-control tends to originate from the failure of engine which makes the aircraft more difficult to control or even impossible to steer it back from abnormal condition. According to [5], 63 fatal incidents were recorded in a study of fatal accidents from 1995 to 2004 where the principal cause was the loss of at least one engine. The failure of engines is usually a major cause of accidents but also the deterioration of the airworthiness can lead to an uncontrollable aircraft.

The above fatal aircraft accidents and others indicate that aircraft upset conditions were caused by, or at least associated with, faults and severe disturbances including jammed control surfaces, loss of engines, icing conditions, wake turbulence, mountain waves, inappropriate control action from the flight crews, loading error, etc. Among those noted above, 10 percent of 837 accidents were triggered by a pilot's incorrect flight control inputs [5, 7], and three were precipitated by the pilot's reaction to component failure and wake turbulence. The unfavorable interaction between pilot's input strategy and the aircraft dynamics can be catastrophic, as stated in the reports [6, 8] by the NTSB, Civil Aviation authority (CAA) and the National Research Council (NRC). The unfavorable aircraft-pilot coupling (APC) may originate from the unusual aircraft characteristics outside the normal flight envelope that are very different from a typical pilot's experience.

As stated by professor B-C Chang: "With hindsight, it is easy to say all these accidents could have been prevented if the maintenance practices had been better to avoid component failures, or if the aircraft had not entered the hazardous regions, or if the flight crews had not made mistakes, etc. It is true that preventive measures should continue to be enhanced. However, it is impossible to eliminate all the faults and disturbing events that may threaten flight safety. Hence, it is necessary that the aircraft be able to respond to unanticipated events and recover once an event has caused it to enter an upset regime."

The aviation community has recognized the necessity of aircraft upset prevention and recovery for almost 40 years. Since May 1970, the NTSB and the CAA have issued more than 10 safety recommendations that addressed training in the recognition of and recovery from unusual attitudes. In July 1995, American Airlines initiated the Advanced Aircraft Maneuvering Program (AAMP) to train its pilots to recognize and respond to an airplane upset [8]. Recently the aviation industry has compiled an airplane upset recovery training aid and posted it on the FAA website [4].

While expressing agreement with the intent of the NTSB and CAA recommendation on the upset recovery training for pilots, the FAA questioned the feasibility of placing a large jet airplane in a nose high, low airspeed, high angle-of-attack situation for training maneuvers. The FAA also did not believe that flight simulators were capable of simulating certain regimes of flight that were beyond the normal flight envelope of an aircraft [8]. Moreover, the NTSB and many aviation safety experts expressed their concerns on the potential negative learning and danger of inappropriate upset recovery simulator training [8]. Due to these concerns and the inability of providing real-world upset recovery experience for the pilots in training, it is difficult to evaluate a pilot's ability for upset recovery at least for now.

The flight dynamics that describe the behavior of the aircraft near or in the upset regime are complex and different from those inside the normal flight envelope, hence they are still not well understood.This leads to the task of designing an aircraft in which prevention and recovery strategies are all embedded. Aircraft upset prevention and recovery is an extremely challenging and formidable task for the pilots because aircraft responses to control commands can be reversed when operating outside of the normal flight envelope. The reaction time from an upset condition is usually very limited, and the pilots need to act quickly and correctly to avoid deepening the crisis. Under these emergency and critical situations, pilots certainly need help from the flight computer/control system to provide the sensing information of all relevant parameters and to detect and accommodate failure, warn or disallow human errors, and perform upset recovery via automated controller reconfiguration or adaptation.

The first step, of course, is to develop a full understanding of unimpaired as well as impaired aircraft,when entering and recovering from LOC situations. This will involve building and verifying aircraft models appropriate for these circumstances including the development of definitions and metrics for identifying when an aircraft is in or near a LOC state, and development of methods to determine if and how such a state can be avoided or recovered from it if it occurs.

1.2 Problem statement and Approach

1.2.1 Problem Statement

The objectives of this thesis is to develop a new methodology for robust reconfigurable control that would prevent the aircraft from entering non - maneuverable domains and recover instantaneously if any unacceptable events (failures of components, pilot mistakes,stall/spin...) occur. Once prevention and recovery control laws are designed, strategies for switching safely between safe maneuverable regimes are embedded and implemented for the entire flight operations. Analysis, implementation and simulation were used for validation of the advanced flight control systems.The overall flight control system can been seen as multi-regimes of operation with reconfigurable controller design off-line for fast actions and better performance. The overall system would be designed and analyzed as Flight Hybrid Fault Tolerant Control Systems(FHFTCS). The picture below shows a sketch of the overall flight control system.

1.2.2 Approach and Limits of previous Approaches

The problem of LOC has been largely ignored in aircraft accident research as opposed to concerns with critical component failures, although they can also contribute to the LOC. Several researchers in control theory have addressed components failures issues using classical methods despite their known limitations. Some of the limitations were associated with the complexity of the problem and associated with not having enough computer tools for online processing. Because optimal decisions could suffer time delay that may be critical for the survivability of the vehicle. Moreover, the online reconfiguration could suffer from insufficient performance of the controller due to systematic change in the system's structure. The inability of the aircraft system to handle disturbance rejection up to the prescribed bounds as thoughts during the modeling process could be catastrophic. For example, Wang et al [Wang:1996] proposed Extended Nonlinear Flight Controllers (ENFC) as an alternative to gain scheduling commonly used in aerospace. They highlight the retention of nonlinear terms in the model while formulating the controller design problem as Hamilton-Jacobi Bellman (HJB)

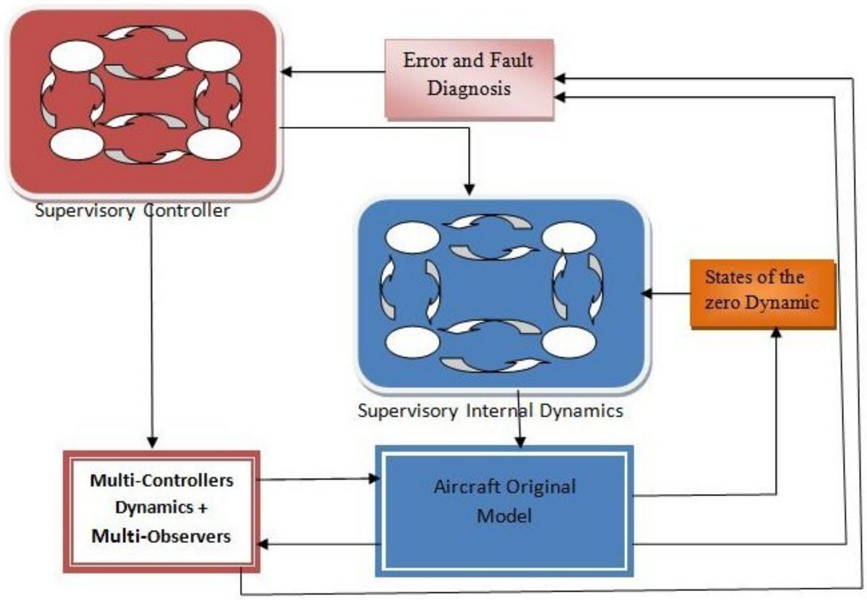


Figure 1.1: Flight Hybrid Fault Tolerant Control System

equation to increase the region of attraction around an operation point. However, the failure cases and critical domains are not addressed and the controllability region is not optimized for safety in switching between modes.

In [12], Multiple Models Switching and Tuning (MMST) is addressed by switching between modes is based on a cost function where the minimum cost decides which controller takes action. One difficulty with this technique, as is the case for Model References Adaptive Control (MRAC) [14, 15], those controllers are functions of estimated states and parameters. On-line estimation is itself a huge problem when large changes in system dynamics are possible. Furthermore, estimation is an inherent problem for aircraft in the event of failure because rapid action must be taken and a time delay in processing may be catastrophic. Moreover, the region of attraction for each mode has not been addressed and also the range of operation for design parameters is not fully explore.

In [16, 21], Kwatny et al address the problem of failures with uncertainty. Each failed mode is associated with a constant disturbance characterizing the severity of the fault. The fault in this case is considered as a time varying disturbance and model as a constant input into the system. A nonlinear disturbance rejection controller is then designed for each fault based on the impaired model. An example is a jammed actuator in which the position of the stuck surface is the uncertainty Even though successful, for certain positions of the stuck deflection surfaces, the regulator did not have enough authority to attenuate all disturbance causes at those specific stuck positions of the deflection surfaces. An alternative would be to switch to a different operating point and operate at reduce performance as illustrated in [23] where the position of elevator is analyzed as a function of velocity.

This thesis addresses the challenges of advanced aircraft control systems where a number of critical problems are explored as shown in figure 1 and described in the next paragraph. From the full understanding of the aircraft's equilibrium manifold to the assessment of envelope protection and recovery using modern approaches. During the control design process, robustness is addressed not just as a measure of the system's capacity to resist, tolerate parameters variations and or disturbances with bounds but also to accept critical failures and continue to operate even at reduced performance. Stability analysis during mode switching is addressed while the admissible range of variable parameters is computed. Stability is ensured with the computation of the guard sets where dual controls, introduced by Feld'baum et al [24], are computed for enabling transitions between modes. Also for instantaneous reset of the functionality and availability of specific devices before allowable actions can be taken. An optimal switching strategy would be investigated to avoid undesirable actions that may worsen the current state of the system.

The increasing demand for highly automated systems that exploit the revolutionary advances in, sensing, computation and communication have enabled control engineers to create increasingly large and complex systems [28]. Along with the growing presence of complex systems in aerospace vehicle we need to be concerned with the capability to accommodate failures in order to complete their safety or mission critical objectives. The design of such systems requires understanding both the discrete dynamics, where logic, switching and coordination of actions are at the core of control, and continuous dynamics, where time evolution and stability are fundamental. Such systems with integrated discrete and continuous dynamics are known in modern control as "hybrid systems".

Understanding the discrete dynamics is essential in moving from one subsystem to another. Issues regarding the stability of the complete integrated system is critical and also reachable sets while enabling switching is essential. Especially, we focus on modeling, design and implementation of aircraft advanced control systems as hybrid fault tolerant systems where the existence of critical failures such as complete loss of actuators which may lead to a completely new dynamical system is addressed. Also the maneuverability near some domains which could lead to a complete dynamical system requiring special strategy for navigation is also addressed.Moreover,we added the option of recovery for post stall or post fault and in doing so it is usually very important to understand the post failure regime or stall before initiating a recovery strategy [26]. Bifurcation theory and continuation methods play an irrefutable role in assessing behavior of dynamical systems [27, 28]. Throughout the process, an investigation of the best switching strategies (supervisor) between modes of the system where modes are conceived from critical failures,maneuverability near critical domain (stall/spin) and environmental induced actions can occur.Switching between modes involve Lyapunov function for stability analysis and reachability analysis to ensure that switching can be initiated. An analysis of each mode is carried out from the computation of low level controller using Variable Structure Control and Extended Linear Quadratic Regulators(HOVSMC,Extended LQR) technique once the safe set is computed.Safe set is obtained by formulating the problem using optimal control and used Hamilton-Jacobi Partial Differential Equation(HJ-PDE) and the safe set is christen maximum controllable set within the initial envelope protection. Prevention in each mode is ensured with optimal control strategies and recovery is also ensured with different control strategies whenever it is necessary.At the end,the overall system is cascaded into an advanced Fly-By-Wire system where implementation, testing and validation are carried out using NASA Generic Transportation Model(GTM).

1.3 Contribution and outline of the thesis

This paragraph presents the contribution to the thesis from the three majors goals that constitute the overall paper: The first important goal deals with aircraft prevention and safe set computation, the second major goal deals with different recovery strategies with their advantages and disadvantages,the final major goals contributes to the formulation of the problem as Aircraft Hybrid Fault Tolerant Control Systems(AHFTCS) where a strategy is implemented for managing the complete advanced flight control system. As contributions we have:

1. An alternative technique for computing the trim condition and aircraft bifurcation curves.

2. The formulation and computation of the maximum envelope protection (safe set) within the predefined envelope.

3. The computation of the controller that protects the maximum allowable safe flight envelope.

4. The computation and implementation of nonlinear smooth controllers for recovery to the normal regime of operation.

5. The design of nonlinear regulators at the bifurcation points (especially in flight control)

6. The computation and implementation of switching controllers for recovery to the normal regime of operation.

7. A new strategy for addressing control magnitude and rate bounds.

8. The formulation and implementation of the overall system christen as Aircraft Hybrid Fault Tolerant System and a strategy for prevention and recovery.

As outlined in this thesis, the first chapter presents the introduction where a survey of aircraft accidents is conducted and the problem statement clearly underlined. The second chapter describes the model used, the computation the bifurcation diagram from a continuation method prospective and from a Quantifier Elimination(QE) technique. The trim condition is also illustrated using quantifier elimination technique. Chapter three covers the prevention with the computation of the safe set and the controller that prevents the aircraft from leaving the set and also the structure of the safe set after failure of an actuator and or a stuck elevator. Chapter four deals with the recovery strategy using nonlinear smooth controllers,chapter five covers the recovery strategy using switching controllers that would restore the flight vehicle within the safe domain while chapter six integrates the overall system into an advanced flight control system using an hybrid control strategy where safety is valuable .

1.4 conclusion

In the introduction, a survey of aircraft accidents was conducted, problem statement of the thesis clearly outlined and the steps for fulfilling the thesis also underlined. With that in mind, we proceed to the next chapter with the description of the model used, the computation of the trim condition, jammed actuators and the bifurcation diagram necessary for the design of a safe flight control system.

2. Equilibrium Points Computation and Aircraft Modeling

This chapter of the thesis focuses on the description of the model used as a tested bet for simulation and the general formulation of the aircraft dynamic equation as six degrees of freedom. In particular,this thesis presents the geometry model of the aircraft in a post stall mode. The particularity is the emphasis on the computation of the equilibrium manifold which plays a significant role in controlling the model in a variety of modes.From the aerodynamic data, the shape of the equilibrium manifold may vary slightly.But based on those aerodynamic data, the input/output structure of the aircraft implies having a complete understanding of the equilibrium points structure and the goal to be achieved by the control system during flight maneuvers. The chapter focuses on the description of the general dynamical equation and techniques for assessing the equilibrium manifold. Moreover it describes the geometrical algebraic position of the aircraft in a post stall mode

2.1 Aircraft Modeling:Euler-Lagrange Equations

Understanding the modeling aspect is essential in controlling any dynamical systems. In general,almost all dynamical systems find their modeling structure embedded in the Euler-Lagrange formulation.The complete dynamical equation is composed of the kinematic equation and the dynamical equation.

$$\begin{aligned} \dot{q} &= V(q)p \\ M(q)\dot{p} + C(p,q)p + F(p,q) &= Q \end{aligned} \tag{2.1.1}$$

where q represents the kinematic coordinates $x, y, z, \varphi, \theta, \psi$, p represents the quasi-velocities u, v, w, p, q, r. $F(p,q)$ represents the vector of aerodynamic external forces which in flight context represents the forces in the x, y, z coordinates and the moments for roll, pitch and yaw L, M, N. Further analytical descriptions of the external forces and moments rely on the aerodynamic experiment where wind tunnel data are extracted, used to generate nonlinear polynomial as a function of the states and controls [29]. After modeling, the above equation can be restructured to fit the general formulation of control system mathematical model.

$$\dot{x} = f(x, u, \mu)$$
$$y = g(x, u, \mu) \tag{2.1.2}$$
$$z = h(x, u, \mu)$$

where $\mu \in \mathbf{R}^q$ is the set of parameters such as the center of gravity location that permit to manage the load distribution in the aircraft, $x \in \mathbf{R}^n$ is the set of states of the given vehicle label $x, y, z, \varphi, \theta, \psi, u, v, w, p, q, r$ and $u \in \mathbf{R}^m$ are the set of controls or deflection surfaces and thrust provides by the type of engine, then $z \in \mathbf{R}^p$ the set of regulated outputs which may vary depending on the performance objectives of the aircraft evolution mode. The $y \in \mathbf{R}^l$ equations (not to be confused with the y-coordinate of the aircraft c.g location) in (2.1.1) represent the position of the sensors necessary for feedback strategies and accurate location of the aircraft in space. Before we move forward, the section below describes the aircraft model uses in this study.

2.1.1 Generic Transportation Model(GTM)

This subsection focus on describing the AirSTAR Generic Transportation Model used by the NASA safety team for validation of technologies that cannot be flight validated with full-scale vehicles. The GTM is a 5.5 % dynamically scaled, generic transportation aircraft, remotely piloted with two power turbo jet engines and includes a collection of sensors, actuators, navigation and telemetry systems.The picture below shows the hardware components and network facilities used by the team for software and hardware validation throughout test and evaluation. The equipment uses by the team involves a remotely pilot aircraft and a ground station where research sits to remotely pilot the vehicle. The attached piece of equipments show the interaction between the ground station and the remotely vehicle which is controlled using the deflection surfaces. Those control surfaces interact with the remote pilot vehicle commands from the ground station. The flight vehicle is also managed through an onboard flight control system and also possess an incredible load factor protection systems.

There exist several types of GTM aircraft but the one that we are dealing with is the T2 and the parameters for the model can be viewed in the Appendix 1 for further details.

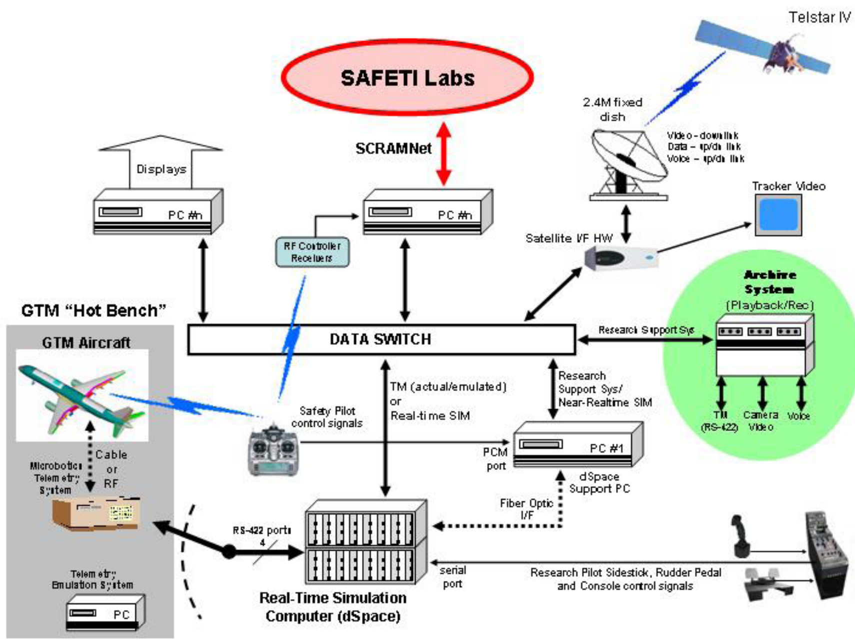


Figure 2.1: GTM-MOS Hardware Systems Overview

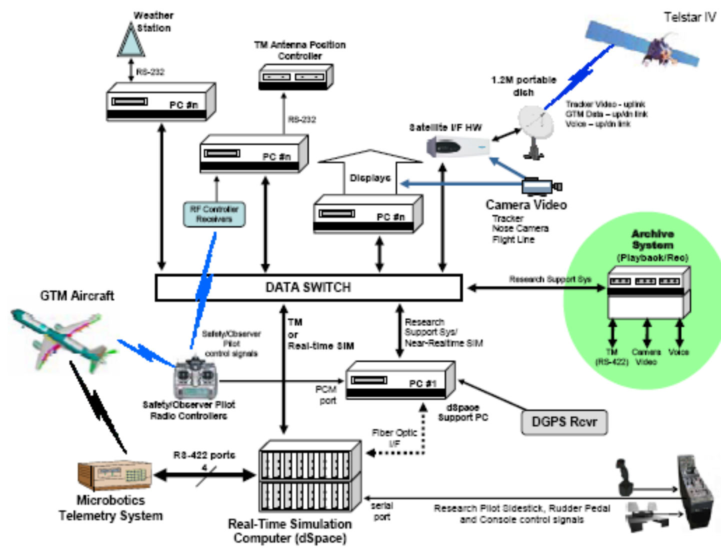


Figure 2.2: GTM-BRS Hardware Systems Overview

2.2 Equilibrium Manifold Computation

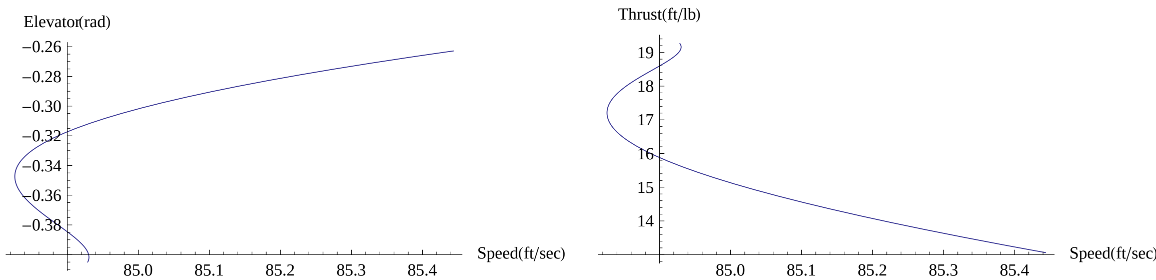
The notion of equilibrium manifold plays a central role in the design of advanced control system because it defines dynamical mode behavior of the aircraft in space. It also underline the design strategy for classical flight controllers design techniques. In general the assumption in studying or designing control system is that the equilibrium manifold is smooth and smooth controller or a sequence of smooth controllers (Gain Scheduling) can be designed to steer the aircraft in the entire state space. It's not always true especially in the context of aircraft that equilibrium manifold has fold regions where solutions to the equilibrium equations are not unique. Computation of those equilibrium is not always obvious because of the algebraic nature of the equilibrium equations and the computer tools available.In Kwatny et al [30], Continuation method is used based on Newton-Raphson and Newton-Raphson Seydel [31] and the solutions can be well approximated. In this section, others techniques are explored despite their limitation in computer tools.

2.2.1 Continuation Method

For decades the continuation method has been the most reliable numerical technique for dealing with aircraft equilibrium equations, especially used to determine trim conditions [32a, 32b, 34, 38]. Although used for qualitative analysis for aircraft dynamics, it is nowadays used as a tool for control system design especially control system near critical regimes.From the aircraft dynamic equations above, the equilibrium equations that map the input/output structure of control system is the following:

$$\begin{aligned} f(x,u,\mu) &= 0 \\ h(x,\mu) &= 0 \end{aligned} \tag{2.2.1}$$

The most important fact that we have in mind when dealing with equilibrium equation is the input/output structure of the dynamical system which completely makes our analysis and design special in addressing the control system design. In general we make the assumption that around a particular equilibrium point,acceptable performance are allowable and approximate linear model is generated and used for controller design purposes. For the continuation method, a particular parameter (center of gravity,Position of the deflection surfaces ...) is identified and used to trace the behavior of the equilibrium surface or curve as the parameter varies within certain compact



(a) ElevatorvsSpeed (b) ThrustvsSpeed

Figure 2.3: Codimension - 1 manifold bifurcation speed curve for GTM longitudinal dynamic

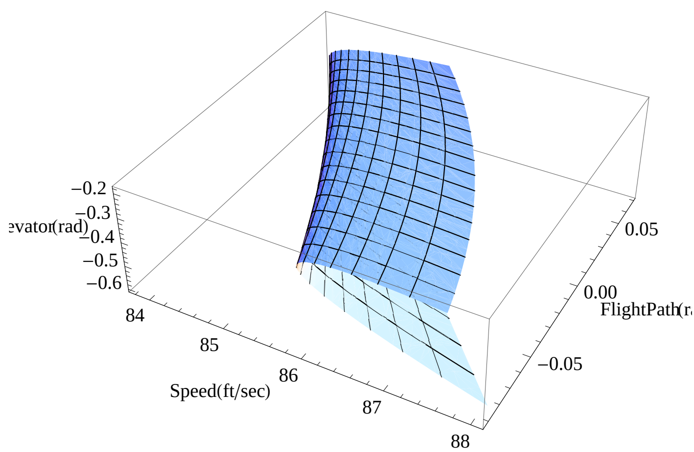


Figure 2.4: Two Dimension Equilibrium manifold bifurcation for GTM longitudinal dynamic

domain. The task is usually reduced to numerical computation for which several existing methods can be used. when the parameter change the equilibrium curve changes and the principal task of a control system (stability, following, regulation, tracking...) can be obstructed. 2.3 shows the bifurcation curve of the longitudinal dynamic of the GTM from a given particular speed and a three dimensional equilibrium surface where speed, flight path angle and angle of attack are slightly perturbed from their equilibrium around there bifurcation point (point at which stall usually occurs).

What makes the bifurcation diagram extremely important especially in the case of flight control system is the fact that at the bifurcation point, aircraft usually stalls followed by spin and pilots in general have difficulties steering the aircraft back to the normal flight regime. Usually the assumption by pilots is that there is a failure at a particular subsystem of the vehicle which may not always be the case. Several facts can be observed from the viewpoint of a control system designer where, it is known [23] that a linearized model a the bifurcation point should exhibit at least one of the following:

1. The aircraft dynamical system has transmission zero at the origin
2. The aircraft dynamical system has an uncontrollable mode with zero eigenvalue
3. The aircraft dynamical system has an unobservable mode with zero eigenvalue
4. The aircraft dynamical has dependent outputs
5. the aircraft dynamical has dependent inputs
6. the aircraft dynamical has structural unstable zero dynamic

At the bifurcation points, at least one of the above facts will be true and the pilot experiences the inability to restore the vehicle and that may be classified in flight as LOC. Because of the sensitivity of the control input near the bifurcation points, it is important to determine with precision the position of the point to better assess the controllability of the flight vehicle.Near the bifurcation points,the zero dynamic of the system will be structurally instable. Although it is true that there are well developed tools (Auto, Consol, etc...) for dealing with computation of bifurcation diagram, it is always interesting to have a complete picture of the bifurcation diagram throughout the compact set of parameters.

2.2.2 Bifurcation Diagram and Quantifier Elimination Method

This section of the thesis highlights application of Quantifier Elimination method to the flight control analysis and design. The technique allows us to view the computation of bifurcation diagram in complete detail and also the compact set of states for which the moment balance can be maintained during flight control orientation. In this particular analysis,we formulate the computation of the bifurcation diagram and the aircraft trim condition using Quantifier Elimination (QE) and obtain the results that were compared with results from existing techniques such as the one used thus far in the field. For details regarding Quantifier Elimination Methods see Dongmo et al [39]. At first we used the above equations (2.2.1) to formulate the computation of the bifurcation diagram for the longitudinal dynamic of the NASA aircraft Generic Transportation Model (GTM) taking advantage of the fact that equations of motion are polynomials function of the states and controls generate from

Wind Tunnel Data. We also know that states and controls of the aircraft are always constrained and the technique is suitable because bounds are taking into consideration during the formulation stage and the design stage:

$$\wp(x) \triangleq \exists u \in U \;\; \exists \mu \in \Omega \;\; [h(x, \mu) = 0 \wedge f(x, u, \mu) = 0 \wedge \chi(x) \wedge \mathfrak{A}(u)] \tag{2.2.2}$$

where

$\chi(x)$ are inequalities that comes from constraint on the states

$\mathfrak{A}(u)$ Inequalities generate from the constraint on the controls

U is simply the compact set of bound controllers.

Ω is the compact convex set of parameters

$\wp(x)$ develops the set of inequalities on the controls for which all inequalities and equalities above are satisfied. The most interesting fact about the formulation above is that, it generates if the solution exist a closed form solution of the equilibrium manifold. The picture below shows a global solution of the equilibrium manifold representing the set of equilibria for all values of the controls and parameters appearing in the moment equations of the GTM dynamics.

Observing the diagram, the shape looks exactly the same as ones obtained with other techniques with the only differences being that it shows a three dimensional view of the equilibrium manifold and projection in any of the two dimensional face gives the curve obtained by using the continuation method.

With advanced technology, orientation of the aircraft with respect to the airflow remains a critical control problems. In wind reference frame, orientation is obtained by measuring instantaneously the value of the angle of attack (α) and the value of the side slip angle(β) [48]. In order to determine the trim condition for which α and β remains constant for some admissible values of the deflection surfaces, we constrained the deflection surfaces within their allowable range.The attitude trim condition is satisfied if the aerodynamic moments acting on the aircraft are equal zero. The problem is formulated using (QE) as follow:

$$\wp(\alpha, \beta) \triangleq \exists u \in \mathfrak{A} \; [C_L = 0 \wedge C_M = 0 \wedge C_N = 0 \wedge \mathfrak{A}(u)] \tag{2.2.3}$$

The picture below shows the compact convex set representing all possible combinations of con-

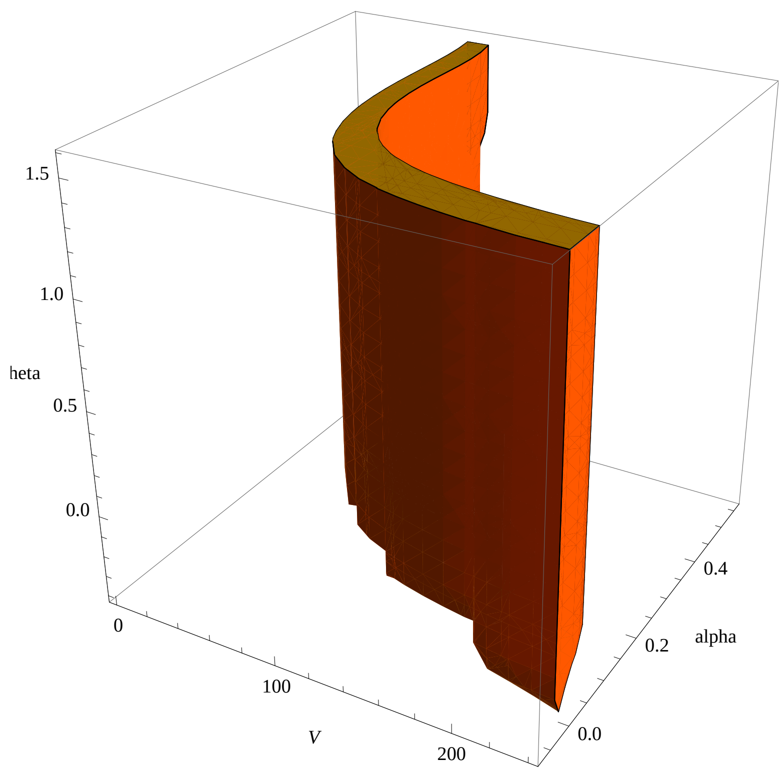


Figure 2.5: QE Global picture of the zeroing equilibrium Manifold for GTM longitudinal dynamic

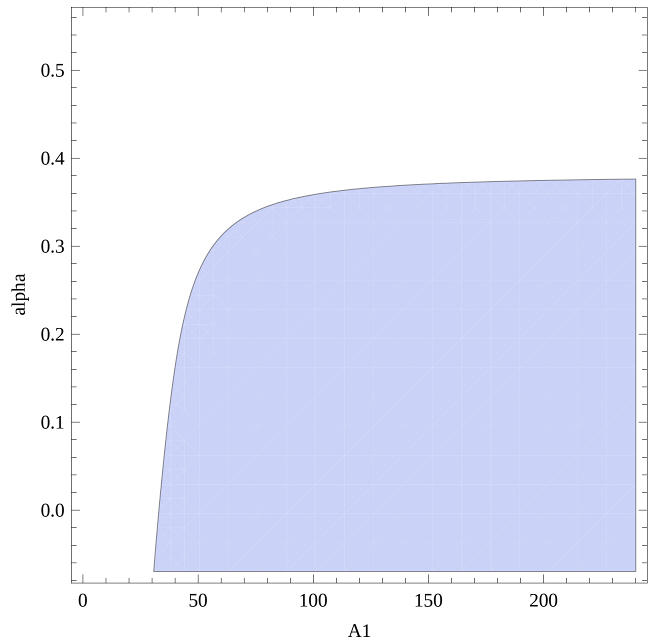


Figure 2.6: QE Global Trim condition for GTM longitudinal dynamic

stant values of α and β for which there exists admissible values of control surface variables.

The advantage of the set obtained is that it is compact and convex and based on those information,other trim values can easily be generated for control system design. Also the region of attraction around the trim condition can also be obtained using the same technique despite its curse of dimensionality.

2.3 conclusion

This chapter covers the description of the model that we use and highlights details of the equations which would very important once failure of actuators would be taken into account.We also look at the equilibrium manifold generated by the continuation methods because our efforts in the design focus on stabilization and regulation in the neighborhood of those critical points. In other words allow maneuverability as much as we can near critical regime. We also investigated different techniques of computing the critical points which is feasible because comparing the results obtained with the previous technique, we matched the results despite the fact the second approach is little more complete.In the next chapter, we looked at the set of departure points to ensures safety for all future time while using different types of controllers adapted along the maneuverability domain.

3. Computation of the Safe Set and Loss - Of - Control Prevention

Safety is the cornerstone of advanced automation in avionics systems. By safety we ascertain that operation within a safe region of the state space can be guarantee for all future time. The question has been raised by several authors in an effort to address prevention of aircraft from departure to an uncontrolled flight regime [32]. Moreover how do we autonomously use generate actions to prevent an aircraft from slipping into an unsafe mode without violating the pilot's rights and what is the structure of the safe region in an any event during the entire flight mission? [41]. It is true that manufacturers and designers have relied on simulation to ensure safety of their design.Failures have shown that some of the unsafe trajectories may be overlooked [107]. An alternative to their approach, we are computing the maximum controlled safe set within the initial flight envelope and the controller that would always ensure safe departure from the set. The question is formulated as a reachability problem where the set of maximum controlled safe set is computed. Computation of the safe set is derived by formulating the problem using (HJ - PDE). Numerical tool based on level set methods [43] can be used to automate the computation of the safe set.

3.1 Computation of the safe Sets

The computation of the safe set plays a significant role in partitioning the state space into regions that are controllable namely safe and uncontrollable regions (unsafe). Especially in constraints systems such as flight control system where the state space is bounds and the controls are also bounds, obtaining the maximum controllable invariant set qualified as safe regions in aircraft control is computationally tractable. Before describing the computation technique and the tools,a few definitions have to be made.

Definition 3.1.1 (**polyhedral**). A set $X \subset \mathbf{R}^n$ is defined as a polyhedral if the boundaries are defined by hyperplane and is bounded if the vertices are finite otherwise it is unbound.

The definition of polyhedral is a general characterization of the constraint state space and any other set can be approximated by the a maximum polyhedral interior to the predefined set. within the initial predefined original set, there is a maximum controllable invariant set (safe set).

Definition 3.1.2 (**Safe Set**). Given a set $X \subset \mathbf{R}^n$ of states and $U \subset \mathbf{R}^m$ of controls, a subset $S \subset X \subset \mathbf{R}^n$ is called safe set if there exists an admissible control such that any trajectory that start in that set remains in that set for all future time.

$$S(t, X) = \{x \in X \subset \mathbf{R}^n | \exists u(\tau) \in U, \forall \tau \in [t_0, t_f] \ x(\tau, t, x, u(\tau)) \in X\}$$

The maximum controllable safe set within the initial envelope is computed by formulating the problem as a reachability problem where the following question can be asked: Given a set of departure states $S \subset X$ what are the possible reachable states in X using a piecewise linear or nonlinear controller for a given time step? The answer to the question is formulated using (HJ-PDE) [42] as follow:

Given an aircraft dynamical system as described in 2.1.1 where the set U of controls is compact and the initial envelope is described by the zero level set of continuous function: $l : \mathbf{R}^n \to \mathbf{R}$ where those functions defined the boundaries of X.

$$X = \{x \in \mathbf{R}^n | l(x) > 0\} \tag{3.1.1}$$

A *safe set* as defined above is the largest controlled reachable set contained in the initial envelope. The solution of the problem requires the following important proposition:

Theorem 3.1.1 (**Viscosity Solution**). *Suppose that $V(x,t)$ is a weak solution of the following terminal value problem.*

$$\begin{aligned} &\frac{\partial V(x,t)}{\partial t} + \min\{0, \sup_{u \in U} \frac{\partial V(x,t)}{\partial t} f(x,u)\} \\ &V(x,T) = l(x) \ over \ \ (x,t) \in \mathbf{R}^n \times [0,T] \end{aligned} \tag{3.1.2}$$

Then

$$S(t, X) = \{x \in \mathbf{R}^n | V(x,t) > 0\} \tag{3.1.3}$$

Proof see [42]

The viscosity solution above can be viewed as the cost-to-go of an optimal control problem where the goal is to derive the control $u(t)$ which maximizes the controllable region within the initial flight envelope while minimizing the value of $l(x(t))$. According to Tomlin et al [107], the cost-to-go can be defined as a unique,bounded and uniformly continuous solution of the following Hamilton-Jacobi

Partial Differential Equation(HJ-PDE):

$$\begin{aligned} H(x,p) &= \min(0, \sup_{u\in U} p^T f(x,u)) \\ \frac{\partial V(x,t)}{\partial t} + H(x, \frac{\partial V(x,t)}{\partial x}) &= 0 \\ V(x,T) &= l(x) \end{aligned} \tag{3.1.4}$$

Notice that the solution of the above HJ-PDE is also a bounded continuous solution of (3.1.4) and the control obtained from solving the Hamiltonian equation above guarantees that all initial states within $S(t,X)$ initiated a trajectory that remains in $S(t,X)$ for all future times. The section below outlines the procedure to compute and the algorithm used in the implementation when automating the general procedure to compute the safe set. From that computation, the controller that would help prevent the aircraft from leaving the safe set and the possibility for recovery if needed is computed.

3.2 Controller for LOC Prevention and Implementation

In our attempt to automate the computation of aircraft safe flight envelope which was outline above as a solution of the HJ - PDE, we first approximated the six degree-of-freedom aircraft dynamic into a set of pairs of equations that represent the different modes of operation in flight dynamical system as suggested by Goman et al [32]. As an example and for illustration purpose, we used the approximated phugoid mode in straight level flight in our analysis and design.We then apply the same technique in other mode such as short period, dutch roll,spiral mode for lateral dynamic even though in certain situation we should investigate the inertia cross coupling effect.

3.2.1 Solving the Hamilton Jacobi Partial Differential Equation

Approach in addressing the computation of the safe set using the Hamiltonian Jacobi-Bellman Equation.The process is outlined below.Solving the HJ - PDE equation is normally done in two major steps:

1. Solving the optimal control and optimal hamiltonian

$$H^*(x,p) = \max_{u\in U} p^T f(x,u) = p^T f(x,u^*) \tag{3.2.1}$$

2. Solve for the cost to go which determine the maximum controllable set(Safe set)

$$\begin{aligned} \frac{\partial V^*(x,t)}{\partial t} &= -H^*(x, \frac{\partial V^*(x,t)}{\partial x}) \\ V^*(x,0) = l(x) &\quad (Boundary\ \ conditions) \end{aligned} \tag{3.2.2}$$

In order to approach the two steps above, we made a few big assumptions which need to be compensated once computation tools become more feasible.

1. The aircraft dynamical system is affine which latter in hybrid context would prove to have limitations

2. The first attempt of solving the HJ - PDE is at the steady state for simple case and for complicated cases,the discrete techniques appears to be useful

3. The initial flight envelope is a polyhedral

$$l(x) = \bigwedge_{i=1}^{n} l_i(x) \ \ where \ \ \frac{\partial l_i(x)}{\partial x} = \pm 1 \tag{3.2.3}$$

The initial flight envelope is defined as a polyhedral representing the outputs to be regulated by the pilot while respecting the constraints on the states and the controls. Taking advantage of the procedure defined by Tomlin et al [45],we do the computation based on each side of the boundary of the initial safe set.The following algorithm is applied for extracting the safe set from the initial flight envelope.

algorithm

1 - Compute $\mathrm{p}_i = \frac{\partial l_i(x)}{\partial x} = [1, 0, ...0]$

$2 - Define\ \ the\ \ optimal\ \ control\ \ associated\ \ to\ \ edge\ \ i$

$3 - Compute\ \ the\ \ \sin gular\ \ po\mathrm{int}\ \ on\ the\ \ edge\ \ i$

$4 - Define\ \ V^i(x,t) = l^i(x(0))\ \ \cos t - to - go\ \ for\ \ the\ \ edge\ \ i$

$5 - \Pr opagate\ \ the\ \ \sin gular\ \ from\ \ edge\ \ i\ \ to\ \ edge\ \ j\ \ backward\ \ in\ time$

$6 - \partial V^i(x,t)\ defines\ \ the\ \ boundaries\ \ of\ \ the\ \ safe\ \ set$

$7 - S(T,X) = \bigcap\limits_{i=1}^{n} (V^i(x,t) \cap \partial V^i(x,t))$ (3.2.4)

The computation of the optimal Hamiltonian shows that there are some points on the boundaries of the safe set which are singular and at those points some of the controls are clearly lost and may require special control action to restore back the aircraft within the normal flight regime. At the same time we can clearly observe the **loss-of- control** near the edge when defining the optimal control. Because the adjoint vector is a binary vector with 1 or -1 at the ith-edge and zero almost everywhere,we can easily determine weather if it is pointing outside or inside. The example below illustrates the algorithm above and is based on the approximated phugoid mode, For more details see Dongmo et al [46].

GTM Longitudinal Phugoid Approximation

We use the reduced longitudinal dynamical equation where a detailed description of the model would be given in the appendix. Here the set of states are $\{V, \gamma\}$ the speed and the flight path angle and the controls $\{T, \delta_e$ are thrust and elevator.The polyhedral associated to the set of states and control is:

$$\begin{aligned} 90 \le V \le 240(ft/\sec) \;\; and \;\; -22 \le \gamma \le 22(\text{degrees}) \\ 0 \le Th \le 30 \;\; and \;\; -40 \le dele \le 20(\text{degrees}) \end{aligned} \tag{3.2.5}$$

The approximated model uses the fact that both the moment balance and the pitch rates should equal zero. Based on those two assumptions, a quasi-static angle of attack α can be derived:

$$\begin{cases} \dot{V} = \frac{1}{m}\{T\cos(\hat{\alpha}) - (1/2)\rho V^2 S C_D(\hat{\alpha}, dele, 0) - mg\sin(\gamma) \\ \dot{\gamma} = \frac{1}{mV}\{T\sin(\hat{\alpha}) - (1/2)\rho V^2 S C_D(\hat{\alpha}, dele, 0) - mg\cos(\gamma) \end{cases} \tag{3.2.6}$$

The boundary of the initial flight envelope which appears to be the set of states that the pilot normally tries to regulate in normal flight using the set of controls is as follows:

$$l(x) = \{V - 90, 240 - V, \gamma + 22, 22 - \gamma\} \tag{3.2.7}$$

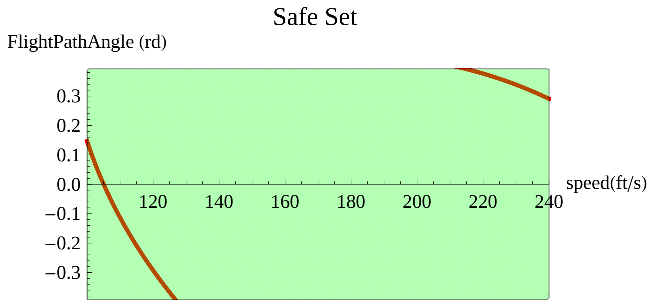


Figure 3.1: Computation of the Safe Set for phugoid mode

The first singular point is obtained by solving the following equation:

$$\{V_{\min}, \gamma_a\} = \{x \in X/V - V_{\min} = 0 \wedge H(p_1, x) = 0\} \tag{3.2.8}$$

At that particular point the flow in pointing inward and the maximum thrust should be used to restore the aircraft back into the normal mode and if the aircraft falls outside the safe set, a particular strategy should be used to generate enough lift that change the direction of flow vector field and point it towards the safe flight mode. A backward interaction is generated from the singular point until the next boundary. The procedure is repeated along the other boundaries and the picture below shows the final Safe set within the initial flight envelope. This particular example is derived with the assumption that the flight path in straight level flight is extremely small almost zero.

The controller below is the least restrictive controller that would maintain all trajectories within the safe set.

$$\begin{aligned}
PreContr = \{&\emptyset \, if \; x \in S \backslash safe\,set \\
&Th \geq Th_{\min}(\gamma), \; if \; (V = V_{\min}) \wedge (\gamma \leq \gamma_s) \\
&Elv = Elv_{\min} \wedge Th = Th_{\max}, \; if \; x \in \partial V_1^a \\
&Elv \leq Elv_{\min}(V), \; if \, (\gamma = \gamma_{\min}) \wedge (V_s \leq V) \\
&Th \leq Th_{\max,s}(\gamma), \, if \; (V = V_{\max}) \wedge (\gamma_S \leq \gamma) \\
&Elv = Elv_{\max} \wedge Th = Th_{\min}, \; if \; x \in \partial V_1^b \\
&Elv \geq Elv_{\max,s}(V), \; if \, (\gamma = \gamma_{\min}) \wedge (V \leq V_S) \\
&U, \; else\}
\end{aligned} \tag{3.2.9}$$

$$Th_{\min}(\gamma) = aV_{\min}^2 + mg\sin(\gamma)$$
$$Th_{\max,s}(\gamma) = aV_{\max}^2 + mg\sin(\gamma)$$
$$Elv_{\max,s}(V) = \frac{m}{bVc}\left(\frac{g\cos(\gamma_{\max})}{V} - \frac{bV(1-c\gamma_{\max})}{m}\right)$$
$$Elv_{\min}(V) = \frac{m}{bVc}\left(\frac{g\cos(\gamma_{\min})}{V} - \frac{bV(1-c\gamma_{\min})}{m}\right)$$

The Least restrictive controller maintains the aircraft within the safe set for any trajectory that starts within that set. The boundaries of the safe set is defined below from direct computation using the fact that the vector field along the boundaries is tangent at any point. An implementation of that trajectory should be carried out to show that the least restrictive controller is effective and can maintain the aircraft within the safe set without overcoming the structural load. Below we show using a two point boundary value problem that in fact recovery is also possible but with different control strategy and those control strategies would be designed in later chapters.

$$\begin{aligned} \partial SafeSet = \{(V,\gamma) | & (V = V_{\min}) \wedge (\gamma_{\min} \le \gamma \le \gamma_s) \vee \\ & (V,\gamma) \in \partial V_1^a \quad \vee \\ & (\gamma = \gamma_{\max}) \wedge (V_s \le V \le V_{\max}) \vee \\ & (V = V_{\max}) \wedge (\gamma_S \le \gamma \le \gamma_{\max}) \vee \\ & (V,\gamma) \in \partial V_1^b \quad \vee \\ & (\gamma = \gamma_{\min}) \wedge (V_{\min} \le V \le V_S)\} \end{aligned} \tag{3.2.10}$$

$$\gamma_s = \sin^{-1}\left(\frac{Th_{\max}}{mg} - \frac{aV_{\min}^2}{mg}\right)$$
$$\gamma_S = \sin^{-1}\left(\frac{Th_{\min}}{mg} - \frac{aV_{\max}^2}{mg}\right)$$

3.3 Computation of the safe set: Jammed Actuators or stuck Elevators

This section adds to the design process, an offline reconfigured model once a jammed actuators is being detected and it also well known from *Suba et al*[16] that not all stuck positions of deflection surfaces uses smooth reconfigured controllers. In this particular approach, we monitored stuck positions incrementally in a sense that a stuck position partition the state space into a safe and an unsafe regions and addresses the smooth reconfiguration offline with respect to a known safe region of the state space. The novelty of this section comes with the computation of the safe set, illustration and then uses the design strategy of the reconfigure controller when a stuck elevator or jammed actuators is been detected [16]. Along with this novelty, we incrementally addressed the impaired safe set with stuck elevator by reducing the range of operations of the allowable deflection surfaces. It's true that in the impaired aircraft case, the stuck elevator ceased to function properly, but in our approach, we can deduced the safe region where the reconfigurable controller should restore the flight vehicle. All this comes with the need of sophisticated control systems where safety requirements and performance goals should be achievable. The outline of this section started with a few approaches to reconfigurable systems, the computations of the safe set associated and finally a reconfiguration strategy that will restore the aircraft within the safe set. The reconfigured strategies restore the aircraft with the knowledge of the deduced impaired safe set. The reconfigured model would be embedded in AHFTCS with knowledge of the impaired safe set.

3.3.1 Hardware Reconfiguration based Redundancy Limit

This particular subsection of the section underlines previous techniques of reconfiguration and their limitations. Then focuses on finding solutions to those limitations first by computing the safe set under failure or stuck elevator within a certain range of operations and second by attempting to examine the safe set if there exists an equilibrium point that can be reached from trajectories which start within the shrink safe set and third manage to reconfigure the unimpaired aircraft that restore the flight vehicle within the impaired safe set. As we would observe, as the aircraft deflection surfaces get stuck at a particular position, the geometry nature of the safe set changes as well. This particular section uses the knowledge of the impaired safe set obtained by reducing the range of operation of the deflection surfaces.

Hardware Reconfiguration based Redundancy Limit

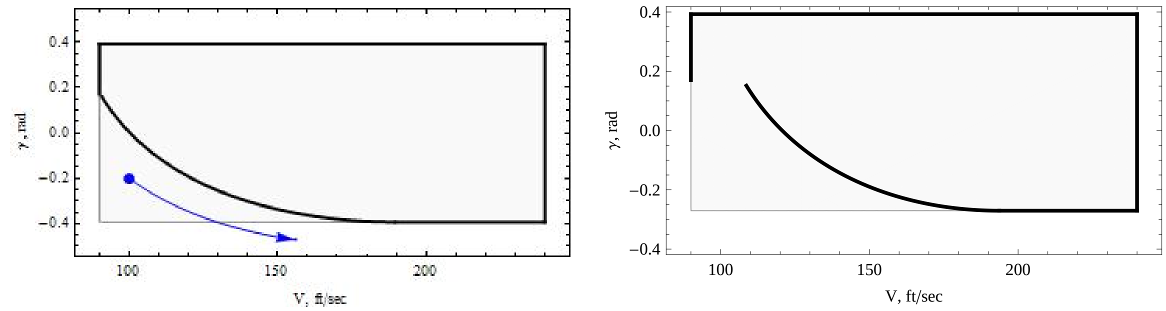
Reconfiguration based redundancy of components (hardware and software) has been at the heart of flight fault tolerant systems for decades. With the development of flight-by-wire,computers have become one of the most critical part of automate advanced flight control system. Because of advanced processing speed, analytical redundancy becomes the subject of major research in the flight community as opposed to hardware redundancy. Also with the advent of big commercial aircrafts and flexible military aircrafts,hardware redundancy becomes less important than the software redundancy [65]. With that in mind,careful measures must be taken into consideration for the optimal reconfiguration of the complete aircraft architecture for better performance and optimal reliability. Thinking in that trends before reconfiguring the system as does Kwatny et al [16], we first compute the allowable safe set within the initial allowable envelope then initiated an offline recovery strategy that restore the aircraft within the compute safe set. Analytical and quantitative measure would be taken to ensure that the reconfigurable aircraft remains in the safe set or can recover to the safe once jammed actuators are observed in the system.

3.3.2 Computation of the Safe Set with Jammed Actuators

In this particular set up, we are using the algorithm outline in the previous section to address the computation of the impaired safe set. Set in which there would always exist a reconfigurable controller that would help us to stay within the maneuverable domain even reduce. At first we use the exact the set up of the previous section with the only difference being that the bounds on elevator control input are reduced either symmetrically or asymmetrically. The reduction is seen here as a stuck position of a deflection surface or a jammed actuator. In our attempt to present the problem, we assume that we have a symmetrically jammed actuators, a meaning of that particular set up is that the deflection surfaces may not be controlled independently as it will be in the asymmetric case. Sofar, we have a set up for the symmetric case which allows us to assess the problem and show what would happen if a particular failure is observed. In this particular set up, we may also determine the controller that allows us to reach the impaired safe set of the aircraft system.

For illustration purposes, we use the simplify model already describe above to compute the impaired reachable safe set. A room for computing the reconfigurable controller is then opened that would restore the flight vehicle within the shrink maneuverable domain obtained by reducing the range of operation of the deflection surfaces.Here no assumption is made with the nominal flight path at straight level flight. For the example below, there may be a scaling issues but enough regarding the model can be found in [46].

An observation can be made by looking at the geometry structure of the unimpaired safe set. It shrinks from outside to inside as the elevator gets stuck from outside to inside as well. Which means that at a particular position, there may be no safe trajectories that can reach the unimpaired safe set. The question that should be raised is what do we do next or how do we manage to maintain the aircraft safe. Further investigation in that direction is a subject for future research.



(a) Nominal Safe Set for reduce Aircraft phugoid Model

(b) Impaired Safe Set with elevator by reducing the range of operation to (-0.456 rd)

Figure 3.2: Unimpaired and Impaired Safe Set Computation with reduced aircraft Model

3.4 Illustration of the Recovery Process Using TPBVP

This section of the chapter stimulate a taste to the notion of recovery. In fact it is true that prevention is possible at the design level using different tools such as stick shocker and others however recovery plays a significant role in the aircraft safety program. Recovery has been used for the most part in the context of reconfiguration where evaluation of the unimpaired aircraft in not even taken into consideration. In this part we assumed the boundary of the safe set is known and the challenge question we are trying to answer refers to can we steer back the aircraft within the normal flight regime after it leaves the flight envelope? The answer to the question is formulated as an optimal control problem.

Based on the approximated dynamical equation (3.2.6) we constructed the associated Hamilton equation as in Markus et al [56].

$$
\begin{aligned}
H(x,p) &= p^T f(x,u) \\
x &\in X \subset \mathbb{R}^n \\
p &\in \mathbb{R}^n (\text{Adjoint } \ vector) \\
u &\in U \subset \mathbb{R}^m
\end{aligned}
\tag{3.4.1}
$$

With the above equation 3.4.1, we compute the optimal control based on one of the major assumptions:<u>the approximated dynamic is affine in control</u> and the optimal Hamiltonian equation is derived.

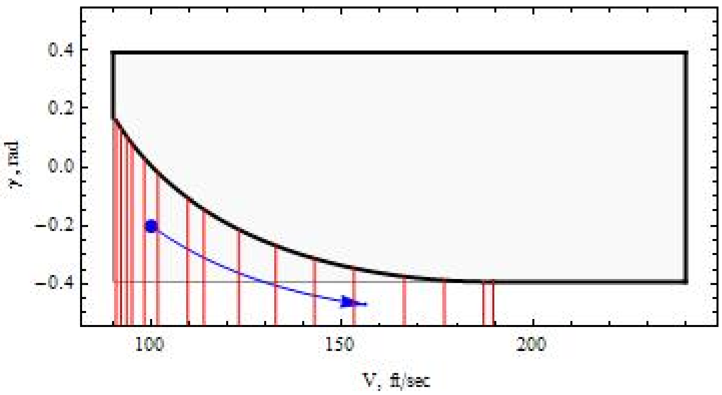


Figure 3.3: Recovery illustration using a Two Point Boundary Value Problem strategy

$$
\begin{aligned}
u^*(p,x) &= Sign(p^T g(x)) \\
H^*(p,x) &= p^T f(x, u^*(p,x))
\end{aligned}
\tag{3.4.2}
$$

From the optimal Hamiltonian $H^*(p,x)$, the adjoint equation is computed and the transversality condition [56] is used to choose initial conditions of the adjoint dynamic along the boundaries of the safe set. Here with the intent to have the terminal condition within the safe region of the state space.

$$
\begin{aligned}
\dot{x} &= \frac{\partial H^*(x,p)}{\partial p} \\
\dot{p} &= -\frac{\partial H^*(x,p)}{\partial x}
\end{aligned}
\tag{3.4.3}
$$

The solution of the equation above shows that once the aircraft leaves the initial flight envelope,a different control strategy can be used to generate enough lift to redirect the flow toward the normal flight envelope. It is possible because within the flight envelope,the trajectory is already using a prescribed set of controllers while outside, a different set of controllers can be applied to maintain flight operation autonomously.

3.5 Conclusion

In conclusion,this chapter outlines and computes the safe set within the initial flight using the HJ - PDE formulation. After the safe Set is computed, prevention is guarantee by computing the least restrictive controller that would assume that all trajectories starting within the safe set remains within the safe set for all future times. In our analysis,a simplified model is used and for a complicated model, Level set methods and Dynamic Programming can be used to tackle the problem. First we used dynamic programming to compute the piecewise linear controller and the Level set method to generate the cost associated with the reachable states. The implementation of the least restrictive controller and the Level set method is an ongoing project. We computed the safe set in this case for a stuck elevator or jammed actuator at a specific position. From the computation of the safe set from a jammed actuator, we observed that the safe set shrinks from outside to the interior as the jammed actuator's angle also decreases. From that computation, we can confirm the fact states by Suba et [21] that at certain stuck position, a smooth reconfigurable may not be appropriated. In that case, only a switching controller may be used to technically reconfigured the flight vehicle. An attempt in that direction was developed in Kwatny et al [108]. Moreover we illustrated the recovery procedure using Two Points Boundary Value Problem (TPBVP) and the satisfaction is that in chapters 4 and 5 , a detail analysis and design of the recovery procedure is laid out. The next few chapters focus on possible recovery strategies designed off-line that would be embedded to the complete Aircraft Hybrid Fault Tolerant Control System (AHFTCS) for advanced flight safety.

4. Aircraft Stall Recovery Using Nonlinear Smooth Regulators Controllers

In chapter 3, prevention and reconfiguration after failure of critical components or in the event of LOC was addressed. For prevention, we solved sequentially the HJ - PDE to compute the safe set also known as the maximum controllable reachable set within the initial flight envelope. Then we derived the control that would keep the trajectory of the flight vehicle on the safe set boundary and to stay within that set for all future time. In a reconfiguration context follow its failure, the impaired aircraft was remodeled, a new equilibrium point computed and the reconfigurable controller design based on the nearest equilibrium point (within the boundary of the safe set). It is certainly true that aircraft reconfiguration seems to attract many researchers but recovery remains one of the most important aspects of aircraft safety and is not fully developed. In this thesis, we consider a number of factors that contribute to aircraft (LOC). Among these, critical trim points in flight are the source of aircraft stall/spin and others more complex behaviors that a pilot would often categorized as LOC. This chapter focuses on evaluating the aircraft post stall behavior and investigates approaches to recovery including their impact on the aircraft structure. Today recovery from a stall/spin depends on the pilot which may not be sufficient because among the leading causes of aircraft accidents, pilot errors are also major factors. Thus in this chapter we consider autonomous control laws that would restore the aircraft back within the allowable maneuverable domain (safe set).

The control laws derived in this chapter are nonlinear smooth controllers obtained from solving HJB equation. The intent is to stabilize the aircraft in the neighborhood of the bifurcation point and regulate to the closest equilibrium point which makes this approach unique compare to other approaches where the dynamical model is perturbed in order to design a particular stabilizing controller. It is true that the obtained controller may saturate and/ or we may not have enough control effort to restore the aircraft back in the safe set.This because near the fold region of the equilibrium surface, critical regions of maneuverability-the zero dynamics of the system are structurally unstable.

The outline of this chapter is as follow: In the first section, we define the aircraft critical motions and connect the notion of center manifold to the recovery process, in section two we evaluate issues that contribute to the aircraft *(LOC)* and the nature of the dynamical motion.Also we describe the different type of behavior encountered in flight dynamical systems. Section three focus on expanding the nonlinear controllers at the bifurcation and what makes it unique in this work. In section four an implementation of the strategy in **Simulink - Matlab** using a composite structure is laid out where observers is added to the model and performance evaluate for better achievements. The contribution of this chapter to the thesis is the design of nonlinear regulators using optimal control for aircraft recovery once in a post stall mode. Altitude drop is evaluated and load factor estimate for validity of the recovery strategy.

4.1 Critical Flight Motions, Center Manifold and Zero Dynamic

4.1.1 Aircraft Critical Flight Regimes

In general, the study of aircraft motion relies on a few factors ranging from an understanding of the structure and geometric nature of the equilibrium points to the augmentation of the aircraft with feedback control laws with the goal of achieving improved flying qualities. The central goal of control designer is to allow the pilot to perform basic tasks (regulating, tracking, following, etc...) safely. Those tasks are usually achievable in certain regions of the flight domain of maneuverability, difficult in others and even thought impossible near other regions and especially at the boundaries of the allowable flight domains (safe set). Because of the challenges facing pilots, automation has become a reliable tool to assist in difficult flight tasks or reducing the pilots load during the flight mission.

While performing the flight mission, a number of unpredictable events can contribute to *(LOC)* such as jammed actuators, structural failures and environmental disturbances. Based on the investigation from Belcastro et al [91], exploration of the aircraft *(LOC)* goes far beyond just the failure of components and others known factors. In doing such analysis, we come to the conclusion that in flight *(LOC)* can be the consequence of maneuverability near the boundaries of the safe set. At the boundaries of those safe set the aircraft maneuvers at High - Angle - Of - Attack where there is not enough control authority to steer the flight vehicle. Within the compute safe set, the flight vehicle may be maneuvering near fold region of the equilibrium manifold where zeroing the outputs using linear quadratic regulators(LQR), the aircraft enters a very sensitive mode because of the unstructured zero dynamics.Moreover, across regions of the state space as thought when augmenting the vehicle with Gain Scheduling [67] , the known control may have limitations. With the above thoughts, investigation of the feedback control laws that would restore the aircraft from a post-stall regime which is associated maneuverability beyond near critical domains. Before going through the investigation of restoring the aircraft, let's survey a number of those critical motions in a post regular mode.

Definition 4.1.1 (Stall). Aircraft in *Stall Regime* is just a condition in which the angle - of - attack exceeds the wings ability to produce lift capable of overcoming the weight and maintain moments balance for the flight vehicle[18, 114, 115,117].

Definition 4.1.2 (Deep Stall [97, 113]). An aircraft in *Deep stall regime* is a mode in which the nose down moment can not be generate with the application of the full nose down motion. Rolling about the velocity vector is a contributing factor to avoid the building of high amplitude of sideslip resulting in a deep stall

Definition 4.1.3 (Wing Rock). Wing Rock motion is characterized by the in flight instability of one of the aircraft's mode especially the Dutch roll mode resulting in the appearance of limit cycle with predominately oscillation in the roll axis [72, 111].

Definition 4.1.4 (Post Stall Gyration [50]). Straight forward definition of the post stall gyration is almost impossible because its motion occurs along the three axis instantaneously and simultaneously. It usually occurs at an angle of attack below the prescribe aircraft's stall AOA.

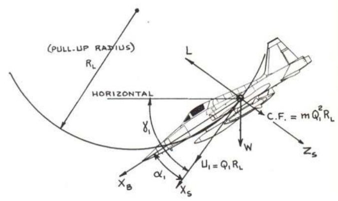
Definition 4.1.5 (Aircraft in Roll Reversal [72, 118]). Roll reversal regime is a situation in which the aircraft rolls opposite to its ailerons inputs resulting in a build up of sideslip angle

Definition 4.1.6 (Aircraft in Falling Leaf Regime). The airplane is first stalled and is then forced into spin. As soon as the spin develops the controls are reversed so that a spin begins in the opposite direction [79].

Definition 4.1.7 (Aircraft in Spin Regime[48, 109, 110, 112]**).**]Spin is just a special type of stall of a worst case of stall if no action is taken to push the nose down and it is characterized by rapid descend and rotation in an helical fashion

Definition 4.1.8 (Departure). Departure can be seen as the event that precipitate the aircraft into the post stall regime where a number of unpredictable behavior can occur: Post Stall Gyration, Spin, Deep Stall, Bucking etc... unsustainable large amplitude response about a certain axis can be clear indication of the departure event [50, 116].

Among those aircraft motions in post stall motion, wing rock with a predominately limit cycle in a roll motion about the x-axis is characterized in mathematic terms by the appearance of Hopf - bifurcation in the nonlinear equilibrium equations; Roll reversal where the aircraft rolls in the opposite direction to the ailerons control input and directional instability when the aircraft depart



(a) Pull Up Aircraft

(b) GTM aircraft in Spin Regime

Figure 4.1: Geometry position of aircraft in a Spin Mode obtained respectively from [19], [1]

from controlled flight and enter a spin mode [48, 36] where the aircraft falls because of not enough lift generation. Two types of spin can be identified: A spin with the nose up known as an upright spin and a spin with nose down known as inverted spin. In either case, aerodynamic modeling becomes very important and cross coupling moment of inertia becomes the known fact and always appears at high angle of attack responsible from the departure to stall and later on enter spin [49, 36].

The position of the aircraft in spin mode as shown by the graph (4.1) allow us to derive a algebraic condition that would be used as a constraint during the recovery process. The algebraic condition uses the fact that in that specific position, the flight vehicle has to rotate within a certain radius to recover to full flight regime. The algebraic condition is established based on the derivation describes in Roskam et al [19].

$$Lift = W + mVq \tag{4.1.1}$$

W is the weight of the aircraft, $q$is the pitch rates and m the mass of the flight vehicle.

4.1.2 Center Manifold and Zero Dynamic

As state above, analysis and design of flight control laws require the restriction of the nonlinear equation to an open local domain where linear approximation is acceptable up to a bound region almost well defined in the neighborhood of the trim point.

Consider the following aircraft dynamical equation:

$$
\begin{aligned}
&\dot{x} = f(x, u, \mu) \\
&y = h(x, \mu) \\
&x \in \Re^n \,;\, u \in \Re^m;\, \mu \in \Re^p
\end{aligned}
\tag{4.1.2}
$$

Given the following control law:

$$
x = k(x, \mu) \tag{4.1.3}
$$

Given the following $x_0.\, u_0, \mu_0$ equilibrium point, the approximated model around that specific equilibrium has the following subsets of eigenvalues that defines the tangent manifold center at the equilibrium point.

$$
\begin{aligned}
&T_0U = E^c \cup\ E^s \cup\ E^u \\
&E^c = \{\lambda \in eig(\partial(f(x, k(x, \mu), \mu)/\partial x)/\ RE(\lambda) = 0\} \\
&E^s = \{\lambda \in eig(\partial(f(x, k(x, \mu), \mu)/\partial x)/\ RE(\lambda) < 0\} \\
&E^u = \{\lambda \in eig(\partial(f(x, k(x, \mu), \mu)/\partial x)/\ RE(\lambda) > 0\}
\end{aligned}
\tag{4.1.4}
$$

Equation (4.1.4) defines the tangent manifold subspace of the $U \subset \Re^{n+m+p}(x_0, u_0, \mu_0)$ neighborhood of the equilibrium. The validity of the approximated model works with the assumption that the equilibrium manifold is smooth and is locally invariant with respect to the vector field. Based on these assumptions, we have the following definition: Let's $U \in V(x_0)$ and $x_0 \in M$ a smooth manifold around the given equilibrium point and is locally invariant and if there exist and local map: $M \Rightarrow \Re^n$ such that the flow is tangent to M.The following situation must hold when the flow function is defined:

$$
W^s_{loc} = \{x_0 \in U/\Psi(t,\, x) \to x_0\ as\ t \to \infty\ \ and\ \Psi(t, x) \in U\ \ forallt \geq 0\} \tag{4.1.5}
$$

and E^c is the fundamental basis of W^s_{loc} [58]

$$
W^u_{loc} = \{x \in U/\Psi(t, x) \to \infty\ as\, t \to \infty\ and\ \Psi(t, x) \in U\ \forall t \leq 0\} \tag{4.1.6}
$$

and E^u is the fundamental basis of W^u_{loc}

Theorem 4.1.1 (Center manifold). *Given the following equilibrium manifold* (x_0, u_0, μ_0) *of the flight dynamical system (4.1.2), a manifold passing through the equilibrium point is the center manifold if it is locally invariant and the tangent space at the equilibrium point is exactly equal to* E^c

In what follows is a natural assessment of the fact that the zero dynamics behave on the center manifold and may be required to be less sensitive and robustly stable in some control problems such as tracking, regulation where the appearance of singularly perturbed zero dynamic is observed. The zero dynamics associated to the system appears with keeping the outputs zero while evaluation the rest of the system.

Theorem 4.1.2 (Existence of zero Dynamics[98]). *Consider the following sets known as invariant sets:*

$$\begin{aligned} &L_0 = h^{-1}(0) \\ &S = \{S_\alpha \subset L_0,\ S_\alpha\, in\ variant\ set\} \\ &M^* = \cup_{S_\alpha \in S} S_\alpha \end{aligned} \tag{4.1.7}$$

If $M^* \neq 0$*, then there exists zero dynamic (resp. sliding dynamic) for the system representing the aircraft equation of motion. In other words, there exists a continuous control such that for all* $x_0 \in M*$ *the flow* $\Psi(t,\, x_0, u(.)) \in M^*\ \forall t \in [0,\, T]$

The definition above clearly highlights the existence of the zero dynamics but it is more clearly seen or compute when the dynamical system is converted into normal form [98]. Before introducing the theorem of the smooth transformation, we highlight the difference between an invariance set (viable set) and the safe set (reachable). In the safe set, the flow always point normal to the set as oppose to the invariant set where the flow is tangent to the set . In computing the safe set we make sure that we have the control that would maintain points in the normal direction because a tangent would mean leaving the set and the opposite is true for the invariant set.

Let's assume there exists zero dynamic that can stabilizable either by smooth controller or switching controller at the point where the model is been analyzed and in our context, we have a fold region at the bifurcation point and the zero dynamics exists and happen to be structurally unstable. Analysis and design would ensure stabilization of the zero dynamic before regulation to a point in the safe set.

Theorem 4.1.3 (Decoupling). *Given the system (4.1.2) we know that under some smooth transformation $(z, \xi) = \phi(x)$, we have the following system*

$$\begin{aligned} \dot{z} &= F(z, \xi, u) \\ \dot{\xi} &= A\xi + p(z, \xi, u) \end{aligned} \tag{4.1.8}$$

And suppose that

$$p(z,\, 0, u) = 0\, and\, for\, all\, z\, near\, 0\, and\, \frac{\partial p}{\partial \xi}(0, 0, 0) = 0$$

If $F(z,\, 0) = 0$ has an asymptotically stable equilibrium point at $z = 0$ and the eigenvalues of all have negative real parts, then the (4.1.2) has an asymptotically equilibrium point. From the above equation, the zero dynamics is defined as:

$$\dot{z} = F(z, 0, u(z, 0)) \tag{4.1.9}$$

Proof can be found in [33]

The above theorem emphasizes the usefulness of the zero dynamics especially near critical domain in flight control. Exploring the behavior of the zero dynamics becomes an increasingly important factor in recovering the aircraft from a post-stall mode to the safe mode within the safe set. It also plays a significant role in preventing the aircraft from departing from controlled flight by sensing the buffeting of the vehicle near stall/spin and taking a necessary control action and recovering once in a post-stall mode. Recovering from a post stall mode is possible whenever we ensure stabilization of the aircraft zero dynamic defined in the above theorem, a hybrid strategy is used to ensure that stabilization and the strategy is also useful in stabilizing the overall system formulated as aircraft hybrid fault tolerant system. The following definition would be essential.

Definition 4.1.9 (Complete Set of matrices[98]). Let's $Z_1 = Z_1^T, ..., Z_N = Z_N^T$ be given symmetric matrices of the same dimension. The collection $\{Z_1, ...Z_N\}$ is said to be complete if for any $x_0 \in \mathbf{R}$ there exists

$$i \in \{1, ..., N\}\ s.t\ \ x_0^T Z_i x_0 \leq 0$$

A collection of those matrices would be strictly complete if for any

$$x_0 \in \Re^n \backslash \{0\}, \exists i \in \{1, ..., N\} \ s.t. \ \ x_0^T Z_i x_0 < 0 \tag{4.1.10}$$

Remark 4.1.1. *It can be shown that if there exists constants reals or binary numbers but using binary number makes it possible to always have the inequality true.*

$$\delta_1 \geq 0, ..., \delta_N \geq 0 \ s.t \ \sum_{i=1}^{N} \delta_i \geq 1 \ and \ \sum_{i=1}^{N} \delta_i Z_i \leq 0 \tag{4.1.11}$$

Then the collection $\{Z_i, \ i \in \{1, ...N\}\}$ is complete. The last inequality is extremely useful because it ensures that there would always exist at least one Lyapunov function for which the system is stable.The following theorem play a central role in the stabilization of the zero dynamic and stabilization of the hybrid system in the last chapter when complete implementation is taken place.

Theorem 4.1.4 (**Zeros Stabilization across switching**). *Consider the nonlinear system (4.1.2) with the following stabilizing controllers:*

$$u_i = K_i(x), \ \ K_i(0) = 0 \ \ \forall i = \{1, ..., N\} \ and \ f(0, \ K_i(0)) = 0 \ \forall i \tag{4.1.12}$$

And suppose that the vector $f(x, \mu, K_i(x, \mu))$ are at least once differentiable and that

$$\dot{x} = \frac{\partial f(x, \mu, K_i(x, \mu))}{\partial x}|_{x=x_0}(x - x_0) + g_i(x, \mu) = F_i(\mu)(x - x_0) + g_i(x, \mu) \tag{4.1.13}$$

Where $g_i(x)$ denotes high order terms satisfying

$$\lim_{||x|| \to 0} \frac{||g_i(x)||}{||x||} = 0, \ \ \forall i = \{1, 2, ...N\} \tag{4.1.14}$$

The aircraft zero dynamics and the entire system is stabilizable by switching between controllers if there exists a positive definite matrix $P = P^T > 0$ for which the set of matrices

$$(F_i^T P + P F_i), \ \ i \in \{1, ..., N\} \tag{4.1.15}$$

is complete

Proof can be found in [98]

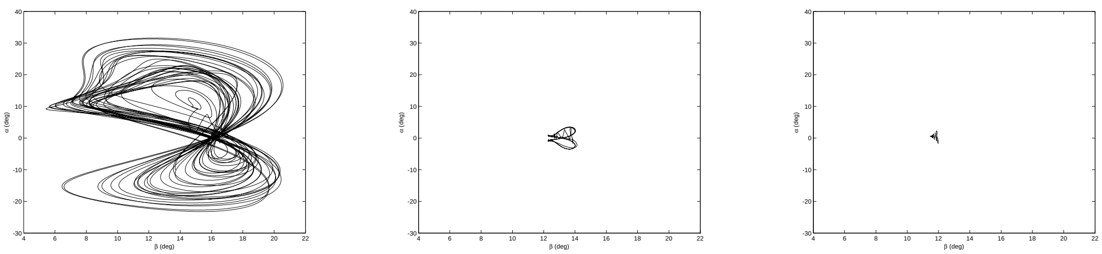
The theorem above plays a significant role in the design of flight control system where equation of motion are by definition non affine in control. The affine approximation appears to have important limitation leading to the *(LOC)* because of crossing line that divide the state space into regions where a single basic controllers can not operate on two adjacent regions. So keeping the system stable across switching region is extremely important and the above theorem is central both dealing with hybrid systems and unstable zeros dynamics. Below are the steps for the zero dynamic stabilization using hybrid strategy:

Algorithm for zero dynamic stabilization.

- Compute the require output zeroing control laws
- Extract the nonlinear zero dynamics
- Linearize the nonlinear zero dynamics
- Apply Theorem (4-2) to determine if the zero dynamics are stabilizable

4.2 Issues and Parameters in Aircraft Recovery Process

A non refutable fact from the flight control literature is that, near critical points the aircraft becomes extremely sensitive to parameters variations [47]. Even though stick shaker can be used as an alarm, the pilot can become overloaded and the aircraft might result in stall. Partitioning the safe set into concrete valid regions where only exist one trim point is an important step, across those regions,the flight vehicle has a structurally unstable zero dynamic. Moreover, the input/ output structure of the flight vehicle near the boundaries of the safe set show a structurally unstable zero dynamic because of the saturation of the deflection surfaces. The sensitivity of the zero dynamic is a contributed factor in the aircraft LOC. Interestingly in the previous approach, LOC appears when preventing designers attempted to force the aircraft to fly near the boundaries [81] where typical control laws may be inappropriate because of the non smooth equilibrium surface of the aircraft due to its fold nature. Because of that particular phenomenon, classical gain scheduled control laws failed Kwatny et al [23]. The problem will be approached differently in this thesis where a motivated goal is to use valid flight control laws in an appropriate region of the state space. In the space of parameters, states and controls, a correlation between important ones such as (alpha-beta) to determine the departure boundaries. One could approach the problem by computing the



(a) trim at 85.5 ft/sec (b) trim at 87.0 ft/sec (c) trim at 90.0 ft/sec

Figure 4.2: Aircraft Behavior as we get close to the bifurcation point

boundaries of the safe set which can also be seen as departure boundaries as was done in previous chapter or performed simulation with a derive Linear Parametric Models (LPV), by varying the parameters, obtained an approximated departure boundaries and or finally performed nonlinear simulation of the flight vehicle as was done below to crack down the departure boundaries. The picture below shows relation between the sideslip angle and the angle of attack at different speeds.

From those observation, an interesting relation can be derived that would serve in the last chapter for making decision and compare with the boundary derived in the first chapter analytically. Holding the angle of attack at a particular trim condition, we can determine the values of beta for which the departure occurs as suggested by [81]. In the plots 4.2, we realized that as we approach the trim bifurcation speed, the aircraft is unstable and one of the following phenomenon can observed.

Two types of spin can be identified: A spin with the nose up known as an upright spin and a spin with nose down known as inverted spin. In either case, aerodynamic modeling becomes very important and cross coupling becomes the known fact and always appears at high angle of attack responsible for the departure to stall and later on enter spin [49, 36]. Although known since the beginning of flight [51], stall/spin recovery till nowadays is manually conducted following a certain number of steps among which:

- Reduce the angle of Attack
- Maintain the aircraft altitude

- Increase Speed

With the advent of bigger commercial aircrafts and modern super maneuverable military aircrafts, certainly recovery will remain an interesting topic for aircraft safety even though recovery is possible but requires high altitude and accurate application of the sequence of steps for success in recovery which means success only with experiences pilots. Despite the sustain training gives to pilots, accidents due to loss of control still important which motivate the idea of designing control systems which autonomously recover from a post stall/spin while following exactly the same steps that a good pilot should perform in real flight. The fact that aircrafts behave differently in a post stall/spin motivate the idea of autonomously restore an aircraft back into the maneuverable domain can result to be more efficient than count on the pilots skills. Before we outlined the section, let's cover a sample algorithm that should be used for validation of the aircraft recovery control system.

Recovery's Algorithm

1. Decrease the Angle-of-Attack by pulling the nose down so the aircraft can regain lift
2. Smoothly increase power which slowly increase speed while maintaining a full coordination of the controls
3. Minimize altitude lost and a perfect recovery procedure must over at most 100fts

In the section below, we elaborated certain control laws that can be used to restore the aircraft back into the normal mode. Throughout the process, we derive nonlinear controllers from an optimal formulation and switching controllers using High Order Sliding Mode Controllers through feedback linearization. The general idea here is to use a hybrid formulation where critical controllers are design offline and embedded into the aircraft for fast action. The advent of digital computers and fly - by - wire control system can make it possible. Before we start with the recovery process, we have to summarize all the recovery techniques use in the thesis where we have from optimal control to High Order Sliding Mode Control through feedback linearization as the table shows. The table also range the technique in terms of altitude drop and the type of model uses for the design.

4.3 Regulation and Stabilization near the Critical Regime Using Optimal Control

In our attempt to master recovery,we developed different scenarios in which a smooth nonlinear controller can be used and if we do not have enough control authority,we have the choice of using

Recovery Techniques	Models	Best technique compare to altitude Drop
Optimal Control	Nonlinear Model	Second
Feedback Linearization	LPV Model	Third
Variable Structure Control(HOSMC)	LPV Model	First

Table 4.1: Aircraft Recovery Techniques uses in the thesis

switching controllers which may be more appropriate than the normal linear smooth controller.

Although it is well known that within the critical regime or near the critical regime smooth controllers may not appropriate, in this section we attempted to used nonlinear smooth controllers first to stabilize the aircraft near the critical regime which is completely different than previous approach [55] because we think in terms of input/output structure instead of using the dynamic to find the controller that stabilize the plant. Comparing the derivation from Garrard et al [11] to whom we borrowed the technique, its approach does not take into consideration the expansion of the model at the bifurcation point and also as oppose to its idea of allowing an expansion of the flight envelope, we are revolving around stabilizing the aircraft near the bifurcation point and restore the aircraft back to the normal mode. Moreover not just stabilizing the aircraft near certain critical regime but also add the output equation for regulation near the critical point. In doing so, we solved the problem in two steps, first attempt to stabilize the aircraft near the bifurcation point by augmenting the aircraft with nonlinear controllers then in the second step investigate the trim point that you can steer into with the available control authority. In what follows is the first step of the design procedure where we used an extended model to compute the controller that would allow us to maintain stability at the equilibrium point.

4.3.1 Stabilization near Critical Flight Regimes

In this section, we used a Taylor approximation model that has the first order terms in the expression where high order terms are added in the state equation and they have real impact as we would realized once we have the solution of the problem. In order to approximate the dynamical equation, we first solving the algebraic equation representing the equilibrium equation as in Kwatny et al [106, 80].

Assume that (x_0, u_0, μ_0) is an equilibrium point of the aircraft dynamical equation(4.1.2) and that it is a static bifurcation point. In general it is particularly difficult to perform feedback regulation near bifurcation points because of the close proximity of multiple equilibria. We used the following state equation obtained from a Taylor expansion of the original aircraft dynamical states in chapter one.

$$\begin{aligned} \dot{x} &= A(\mu)x + B(\mu)u + \phi(x,\mu) + hot \\ y &= g(x,\mu) \\ e &= h(x,\mu) \end{aligned} \tag{4.3.1}$$

$$\phi(0,\mu) = 0 \tag{4.3.2}$$

Our final goal is to design an optimal stabilizing state feedback regulator which will ensure zeroing the error equation.

Remark 4.3.1. The separation principle allows us to deal with these equations separately i.e. controller design and observer design. We would analyze the problem in two steps:

- Stabilization of a predefined manifold
- Regulation of key variables in flight

Achieving regulation is usually at the expense of a certain cost function associated with the control function $u(x)$ and the states of the system.The minimization of that cost allow us to maintain regulation once reaching the zeroing manifold.

The system of equation (4.1.14) at a given equilibrium point(x_0, u_0, μ_0) is stable if exponential stability of the closed loop system holds in the neighborhood of μ_0 characterized by $u_0 = k(x_0, \mu_0)$ In the case state feedback and $\eta(\hat{x}^*, y) = u^*$ In the case of dynamic feedback [67].

Before we attempt to design the stabilizing state feedback, let's state the following result which enforces the stability near the perturbed equilibrium point under the parameter variation which is generally the case near stall in flight control.

Theorem 4.3.1 (**Stability**). *Let's suppose that $(x^*, \hat{x}^*)$ denotes an exponentially stable equilibrium points of the closed loop dynamics at $\mu = \mu^*$ then there exists a function*

$$x_c(\mu) = (x(\mu), \hat{x}(\mu) \; with \, x_c(\mu^*) = (x^*, \hat{x}^*) \tag{4.3.3}$$

such that the composite system satisfied the following:

$$f_c(x_c(\mu), \mu) = 0 \; where \; (f_c(x_c(\mu), \mu) \equiv observer + state\, equations) \tag{4.3.4}$$

The concept of exponential stabilizability and detectability are then used to enforce the design of the compact system as it should be the case near the aircraft stall point.

Definition 4.3.2 (**Exponentially Stabilizability**)**.** The extended state equation above [67]

$$\dot{x} = A(\mu)x + \varphi(x, \mu) + B(\mu)u; x \in \Re^n; u \in \Re^m; \mu \in \Re^s \tag{4.3.5}$$

with

$$A(\mu^*)x^* + \varphi(x^*, \mu^*) + B(\mu^*) = 0 \tag{4.3.6}$$

is exponentially stabilizable at $(x^*(\mu^*), u^*(\mu^*))$ if there exists a function $u = k(x, \mu)$ defined on the neighborhood of the equilibrium point with $u^* = k(x^*, \mu^*)$ so that the equilibrium point $x^*(\mu^*)$ of the closed loop system is exponentially stable.

Definition 4.3.3 (**Exponentially Detectability**)**.** The extended state equation above(5.4.1)with the output equation

$$y = g(x, \mu), y \in \Re^q \tag{4.3.7}$$

with $A(\mu^*)x^* + \varphi(x^*, \mu^*) + B(\mu^*) = 0$ and$y^*(\mu^*) = g(x^*(\mu^*))$ is exponentially detectable at $(x^*(\mu^*), u^*(\mu^*))$ if there exists an observer equation

$$\dot{\hat{x}} = \gamma(\hat{x}, y); \; \hat{x} \in \Re^n \tag{4.3.8}$$

Where the observer flow function is defined in the neighborhood of $(x^*(\mu^*), y^*(\mu^*))$ in $\Re^n \times \Re^q$ and the following conditions are satisfied:

$$\gamma(\hat{x}^*(\mu^*), y^*(\mu^*)) = 0 \, and \, \gamma(x(\mu), g(x(\mu)) = f_e(x(\mu), u^*(\mu^*)) \tag{4.3.9}$$

And the point $x^*(\mu^*) = \hat{x}(\mu)$ is an exponentially stable equilibrium point of the system

$$\dot{\hat{x}} = \gamma(\hat{x}, y^*(\mu^*)) \tag{4.3.10}$$

The design of dynamic stabilizing state feedback controller is durable in this context if the Taylor approximation model to the high order obtained at the bifurcation point is exponentially stabilizable and exponentially detectable at the require point and should be a condition for stop while increasing the nonlinear terms in the model. Usually because of the close proximity of equilibrium, the nonlinear term added in the model help quench the limit cycle that would have started if the system was approximated by a linear model.

Optimal state feedback near Flight Critical Regimes

In order to design an optimal nonlinear stabilizing controller, we formulated the problem as *Hamilton Jacobi - Bellman (HJB)* equation and solve the steady state equation by writing the solution as series expansion and truncated it at a reasonable high order terms. In order to approach the problem, we make the assumption that the system is stabilizable in the Lyapunov sense:

Theorem 4.3.2 (Direct Lyapunov Stability Theorem). *Given the aircraft dynamical system describe in (4.1.2), if there exist a positive-definite continuous Lyapunov function $V(x)$ with $V(0) = 0$ with continuous first partial derivatives with respect to the state. The equilibrium state is stable if*

$$\frac{dV(x)}{dt} = (\frac{\partial V(x)}{\partial x})^T \frac{dx}{dt} < 0 \;\; and \;\; V(x) \geq 0 \tag{4.3.11}$$

Once we observe the previous theorem, it can be connected to the *Hamilton - Jacobi Bellman Equation (HJB - E)* by setting and optimal control cost to be minimized while solving for the controller gain:

$$\begin{aligned} J(x(.), u(.)) &= \int_{t_0}^{t_f} (\Delta x^T Q \Delta x + \Delta u^T R \Delta u) dt \\ Q = C^T C \geq 0; \;\; R > 0; \;\; \Delta x &= x - x_0; \Delta u = u - u_0 \end{aligned} \tag{4.3.12}$$

We write the Lyapunov function as a series expansion and also write the control effort as a function of expand Lyapunov series as it is extended.

$$V(x) = \sum_{n=0}^{\infty} V_n(x) \;\; and \;\; \Delta u(x) = -R^{-1} B^T \sum_{n=0}^{\infty} \frac{\partial V_n(x)}{\partial x} \tag{4.3.13}$$

Substituting the control above into the cost (4.3.9) and differentiating the series Lyapunov function and putting equation (4.3.9) and (4.3.10), the resulting equation to be solved sequentially with the assumption that at each sequence, the Lyapunov function is quadratic and can be extended to nonquadratic for better performance. An intermediated step before the resulting equation is to construct the Hamilton Jacobi Bellman Equation from which the Hamilton-Jacobian Bellman is derived with necessary conditions satisfied.

$$H(\Delta x(t), \Delta u(t), \frac{\partial V(x)}{\partial x}) = \Delta x^T Q \Delta x + \Delta u^{^T} R \Delta u + \frac{\partial V(\Delta x)}{\partial \Delta x}\dot{x} \tag{4.3.14}$$

Minimizing the equation above, we obtained the controller given above and the resulting steady-state *Hamilton Jacobi Bellman Equation (HJB)* is given below obtained from the following generalized equation.

$$\frac{\partial V(\Delta x)}{\partial t} + \Delta x^T Q \Delta x - (\frac{\partial V(\Delta x)}{\partial \Delta x})^T B R^{-1} B^T (\frac{\partial V(\Delta x)}{\partial \Delta x}) + (\frac{\partial V(\Delta x)}{\Delta x})^T (A\Delta x + \varphi(\Delta x)) = 0 \tag{4.3.15}$$

The resulting equation below is obtained from the equation above while substituting the approximated Lyapunov series and is then solved sequentially as a result of the following theorem.

$$\frac{\partial V(\Delta x)}{\partial t} + \Delta x^T Q \Delta x - (\frac{\partial V(\Delta x)}{\partial \Delta x})^T B R^{-1} B^T (\frac{\partial V(\Delta x)}{\partial \Delta x}) + (\frac{\partial V(\Delta x)}{\Delta x})^T (A\Delta x + \varphi(\Delta x)) = 0 \tag{4.3.16}$$

Theorem 4.3.3 (Optimality). *Given the above aircraft dynamical equation (4.1.13), the approximate series Lyapunov quadratic positive C^k function:$V(.) : \Re^n \to \Re^+$ is the solution of (4.3.4). The nonlinear control (4.3.2) generating the solution: $x(x_0, t) : [t_0,\ t_f] \to \Re^n$ is optimal if sufficient conditions of optimality are satisfied.*

$$\begin{aligned} &(\tfrac{\partial V_n(x)}{\partial x})^T Ax - \tfrac{1}{4}\tfrac{\partial V_n}{\partial x} BR^{-1}B^T \tfrac{\partial V_0}{\partial x} - \tfrac{1}{4}\tfrac{\partial V_0}{\partial x} BR^{-1}B^T \tfrac{\partial V n}{\partial x} + (\tfrac{\partial V_0}{\partial x})^T \phi_{n+1} \\ &+ \sum_{k=0}^{n} (\tfrac{\partial V_k}{\partial x})^T \phi_{n+1-k} - \tfrac{1}{4}\sum_{i=1}^{n-1} \tfrac{\partial V_k^T}{\partial x} BR^{-1}B^T \tfrac{\partial V_{n-k}}{\partial x} = 0 \quad k = 0, \ldots\ldots\ldots\ldots\ldots n \end{aligned} \tag{4.3.17}$$

Solving the equation above up to certain degree allow us to obtain an impressive and optimal nonlinear robust stabilization controller with the parameterizing Lyapunov function as shown below and solve for the algebraic parameters and the solution is given below.

$$V_n(x) = \sum_{k=0}^{n+2-k} \sum_{j=0}^{n+2} a^n_{n+2-j-k} x_1^{n+2-j-k} x_2^j x_3^k \tag{4.3.18}$$

Expanding the Lyapunov function up to a certain order decide by the designer makes the computation a little more demanding but if instead the order is chosen reasonably, the computation makes it extremely easy to solve. The example below illustrates, the application of the above procedure with the GTM longitudinal dynamic expands at the bifurcation and the input to state is used to derive the nonlinear stabilization instead of using the dynamical approach as others approaches. Figure 1 shows the control effort to stabilize the aircraft at a reasonable altitude. Further investigations are actually conducted sofar to steer the aircraft near at a much more reasonable equilibrium point that belongs to the safe set.

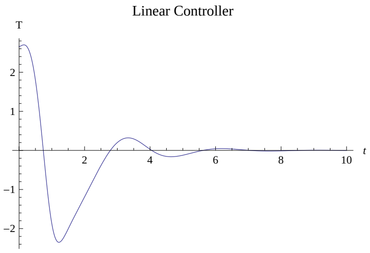
Using the Generic Transportation Model Aircraft

The GTM example that we used here is an Extended Linearize Model at the bifurcation point that we actually stabilize the perturbed model using an optimal nonlinear controller. The program for stabilization is carefully explain and how it runs and can be extended for a full six degree of freedom aircraft. The model is defined below

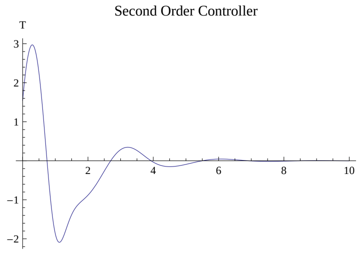
$$\begin{aligned} &trim\,Point \equiv \{V = 84.826\,ft/sec; alpha = 0.3055, theta = 0.3055, q = 0\} \\ &Controller \equiv \{Th = 17.2055707; dele = -0.34737048\} \end{aligned} \tag{4.3.19}$$

The high order nonlinear states added into the system is defined below:

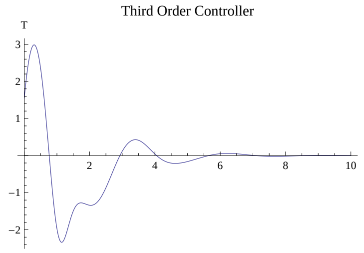
$$\begin{aligned} &f_2 = \{-0.002023 * x1^2 - 2.572231x1 * x2 + 227.15369x2^2 \\ &-0.05177 * x1 * x4 - 34.378x2 * x4 - 0.22053 * x4^2, \\ &0.00007278 * x1^2 - 0.0213259 * x1 * x2 - 21.9029 * x2^2 + 0.584719 * x2 * x3 \\ &-0.2923 * x3^2 - 0.6078 * x2 * x4 + 0.0082 * x4^2, 0, 0.0005586 * x1^2 + 1.120259 * x1 * x2 \\ &+601.95259 * x2^2 - 3.547 * x3^2 - 3.12158 * x1 * x4 - 959.3777 * x2 * x3\} \end{aligned}$$



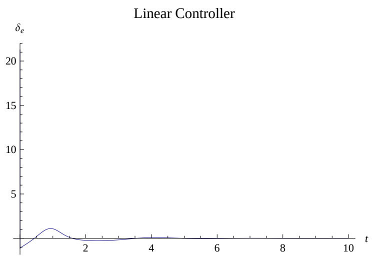
(a) Linear Thrust control law



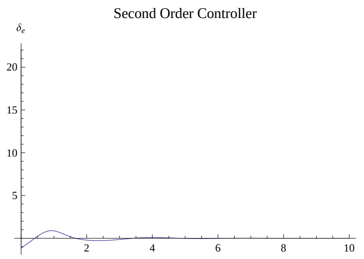
(b) Second Order Thrust control law



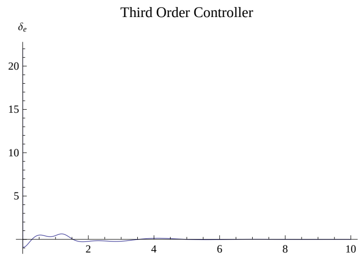
(c) Third Order Thrust control law



(d) Linear Elevator control law



(e) Second Order Elevator control Law



(f) Third order Elevator control law

Figure 4.3: Nonlinear smooth feedback control input with third order Taylor approximation during the recovery process

$$f_3 = \{0.0151618 * x1^2 * x2 - 5.54919 * x1 * x2^2 - 2796.95 * x2^3 - 24.7997 * x2^2 * x3 + 24.7997 * x2 * x3^2$$

$$-8.26657 * x3^3 + 0.405286 * x1x2x4 + 106.685 * x2^2 * x4 - 0.0695612 * x2x4^2,$$

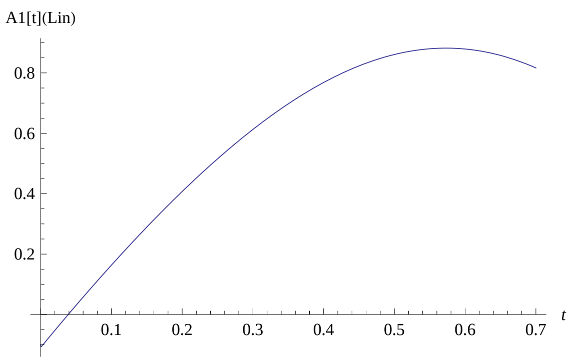
8.580223220071474*^(-7)*x1^3

$$+0.0000268835 * x1^2 * x2 + 0.252036 * x1x2^2 + 48.7813x2^3 + 0.00689317 * x1x2 * x3 - 0.00344658x1 * x3^2$$

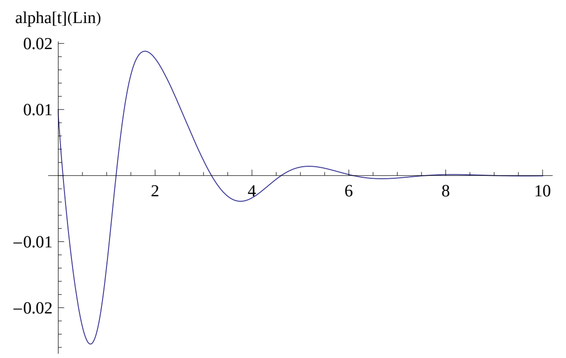
$+1.02865 * x2^2x4 + 9.667382198437735$*^(-6)*x1x4$^2 - 0.00259978 * x2x4^2, 0,$

$$-0.00660328 * x1^2 * x2 - 14.1926 * x1x2^2 - 1155.0 * x2^3 - 0.37301 * x3^3 + 11.3099 * x1x2x4 + 93.2967x2^2x4\}$$

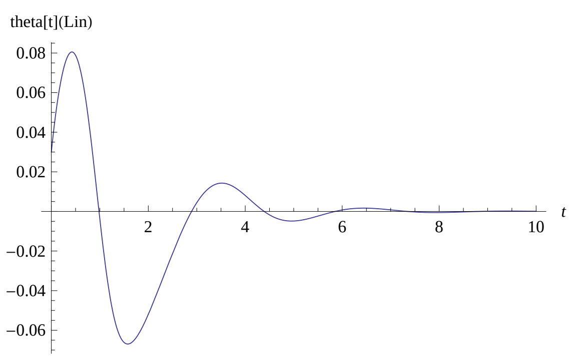
The results from optimal controller design at the bifurcation point with nonlinear terms added into the system. From the design of linear optimal control law to the third order nonlinear control laws.



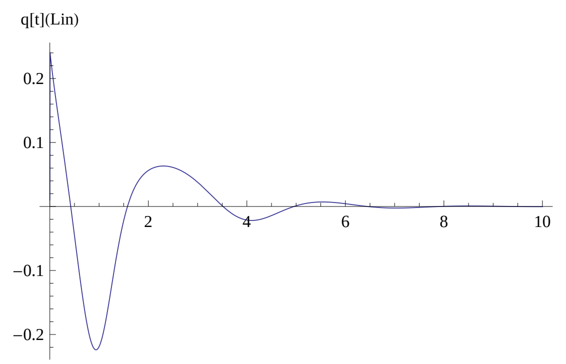
(a) Speed of second order approximation



(b) Angle of attack of second order approximation

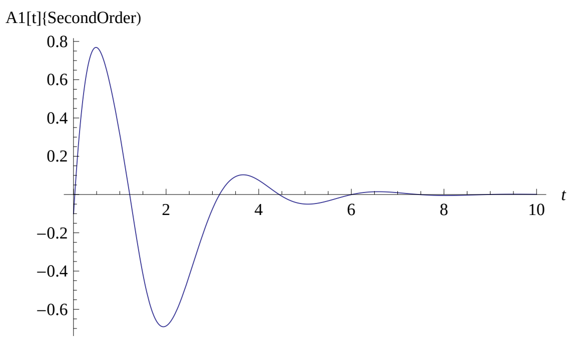


(c) Pitch Angle for the second order Approximation

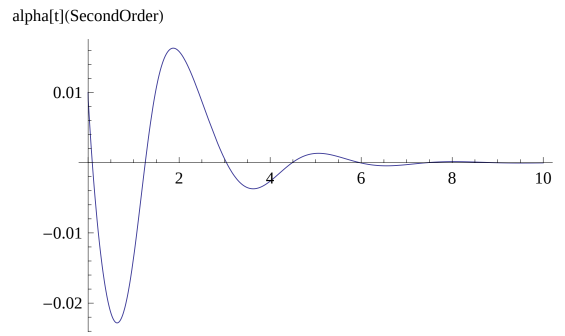


(d) Pitch rate for the second order approximation

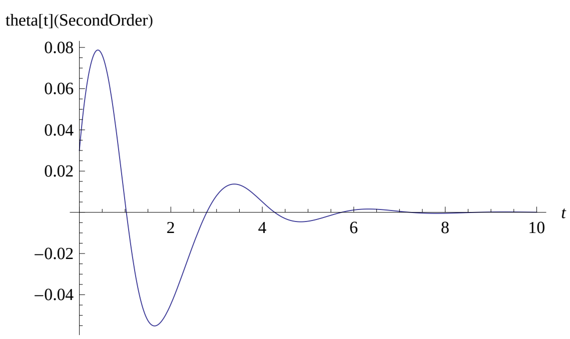
Figure 4.4: Aircraft Longitudinal response to the third order approximation during the recovery process using Linear controller



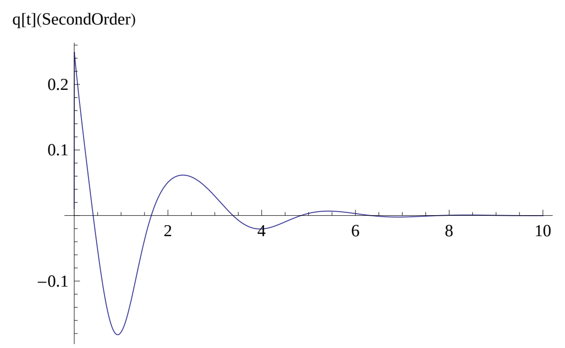
(a) Speed of second order approximation



(b) Angle of attack of second order approximation

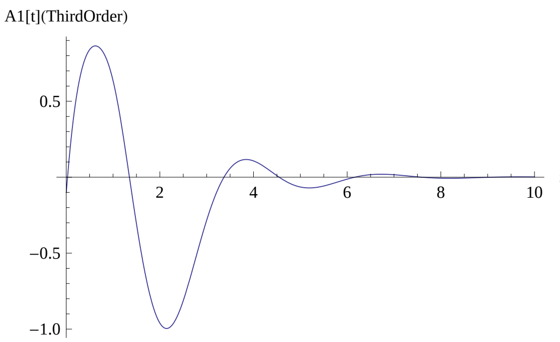


(c) Pitch Angle for the second order Approximation

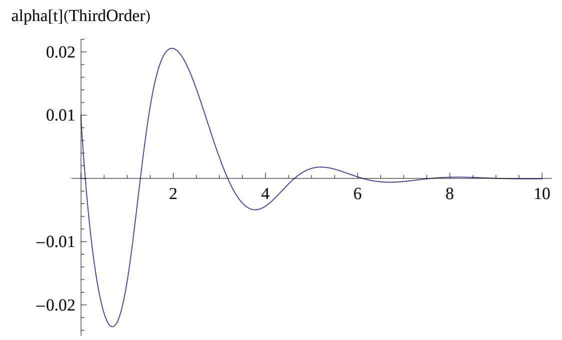


(d) Pitch rate for the second order approximation

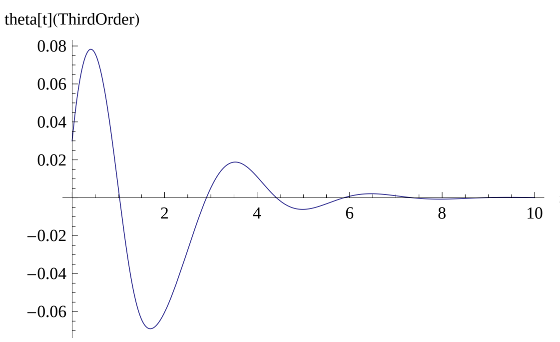
Figure 4.5: Aircraft Longitudinal response with a third order Taylor approximation during the recovery process using a second order controller



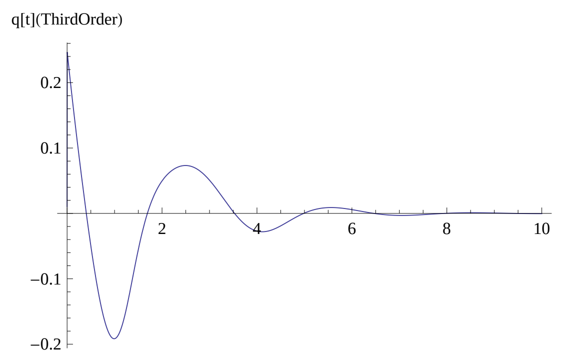
(a) Speed response of the third order approximation



(b) Angle of attack response of the third order approximation



(c) Pitch angle response of the third order approximation



(d) Pitch rate response of the third order approximation

Figure 4.6: Aircraft Longitudinal Response with a third Order Taylor Approximation during the recovery process using a third order controller

4.3.2 Aircraft Regulation near the Critical Regime

Regulation is an extremely important problem in control system design and especially in flight control because pilot usually attempts to regulate key components during flight maneuver. Information are not always available and we obtained those information through observers [89]. In this particular section of the chapter, we are expanding what was done previously by trying to restore the aircraft in the safe set computed for prevention in the previous chapter. The problem is how far can we restore the flight vehicle back into the safe set. Here, we compute the feed forward gain as suggest by Kwatny et al [106, 16] where the state is written as a function of the reference input and cascade with the stabilization controller computed above. Before we proceed, let's state the output zeroing theorem which is necessary for output regulation

Theorem 4.3.4 (**Regulation**). *Given the original system (4.1.2) augmented with its observer equation states below*

$$\dot{\hat{x}} = \chi(\hat{x}, y) \ \textit{and} \ u = \kappa(\hat{x}, y) \tag{4.3.20}$$

an equilibrium point $(x_0, \, \hat{x}_0)$ *is exponentially stabilizable and detectable in the neighborhood of* $\mu = \mu_0$. *Then there exists a function* $\bar{x} = x(\mu)$ *and* $\bar{\hat{x}} = \hat{x}(\mu)$ *in the neighborhood of* $\mu = \mu_0$ *such that the following equation holds:*

$$\begin{aligned} &f(\vec{x}, \kappa(\bar{\hat{x}}, h(\bar{x}, \mu)), \mu) = 0 \\ &\chi(\bar{\hat{x}}, \, h(\bar{x}, \mu)) = 0 \end{aligned} \tag{4.3.21}$$

Theorem 4.3.5 (**Zeroing Manifold**). *Given the original system (4.1.1), and* x_0 *a solution of the equilibrium equation and* U *a neighborhood of that point. Let's* M *a smooth manifold that contain* x_0 *is* local invariant *if there exists a smooth map* $u : M \to \mathbf{R}^m$ *such that the flow is tangent to* M *for all* $x \in M$.

M is called *output zeroing manifold* and ascertain the regulation of certain output variables.

Remark 4.3.2. :
In our case, near the boundary or before reaching the boundary, the output zeroing manifold is sharp such that the existence of such unique $u : M \to \mathbf{R}^m$ may or may not be possible. Nonlinear controllers or switching controllers may be appropriated for either stabilizing and steering the flight

vehicle back to the normal mode.

Achieving regulation is obtained by making the algebraic equations below hold at steady state. The process is defined below.

Given the following system:

$$\begin{aligned} \dot{x} &= f(x, u, \mu, w) \\ \dot{w} &= s(w) \\ e &= h(x, \mu, w) \end{aligned} \tag{4.3.22}$$

The difficulties with such manifold is sometimes that the map may not be unique which makes the problem extremely difficult and also such map may not exist because of the fold equilibrium manifold near the equilibrium point. Moreover based on the implicit function theorem, such map may not exist or it may be too difficult to restore the aircraft near such points. In this section, we assume that we can restore the aircraft in the neighborhood of that equilibrium point. The compute safe set give us an inside of how to pick the equilibrium point before attempting to steer the aircraft. Regulation is achieved at steady states and let set up the steady state algebraic equation. With these assumptions: The system above is detectable and the actuator dynamic is stable the we used the following theorem to exercise the computation of regulating function once we reach the steady state.

Theorem 4.3.6 (**Output Regulation**). *The output Regulator problem is solvable if and only if there exist mappings $x = \chi(w, \mu)$ and $u = a(w, \mu)$ with $\chi(w, \mu) = 0$ and $a(w, \mu) = 0$, both defined in the neighborhood of the origin satisfying the conditions*

$$\begin{aligned} f(\chi(w, \mu), a(w, \mu), \mu, w) &= 0 \\ h(\chi(w, \mu), \mu, w) &= 0 \end{aligned} \tag{4.3.23}$$

The above equations are solvable for the regulating functions$\chi(w, \mu) and a(w, \mu)$ using Taylor series around the neighborhood of the input signal from pilot command which become constant disturbance at steady states.

$$\chi(w,\mu) = \chi'(0,\mu)w + \chi''(0,\mu)w^2 + ...$$
$$a(w,\mu) = a'(0,\mu)w + a''(0,\mu)w^2 + ... \tag{4.3.24}$$

The final output regulation problem is solvable if there exists if there exists a stabilization function $k(x,\mu)$ and an observer function$\Theta(\hat{x}(\mu), y(\mu))$ such that the output regulator has the following solution

$$u(x,\mu) = a(w,\mu) + k(x,\chi(w,\mu))$$
$$\dot{\hat{x}} = f(\hat{x}(\mu), u(\mu)) + \Theta(\hat{x}(\mu), y(\mu)) \tag{4.3.25}$$

where the following conditions hold:

1. $k(x,\mu)$ is a state feedback that exponentially stabilize $\dot{x} = f(x(\mu), k(x(\mu)), 0)$

2. $\Theta(\hat{x}(\mu), y(\mu))$ is any function that satisfies the following conditions $\Theta(0,0) = 0$ and $\Theta(\hat{x}(\mu), g(x(\mu))) = f(x(\mu))$ and also the origin exponentially stable for the reduce dynamic equation $\dot{\hat{x}} = \Theta(\hat{x}(\mu), 0)$

Before we finalize the design of the optimal control law for the GTM recovery process, we have to acknowledge a few assumptions and a theorem that makes it possible to design an optimal regulator in the neighborhood of the bifurcation points.

Assumptions[23]

1. Steady state algebraic equations above is solvable if we have at least as many outputs as there exists the same number of inputs.

2. The system $\dot{x} = f(x, u, \mu_0)$ is exponentially stabilizable at (x_0, u_0)

3. The aircraft system of equations is exponentially detectable at (x_0, u_0, μ_0)

   The third assumption would allow the design of a local observer such that the following equation $||x(t) - \hat{x}(t)|| \to 0 \; as \; t \to \infty$ holds in the neighborhood of the equilibrium point (x_0, u_0, μ_0)

Example*Implementation of the regulator (NASA Aircraft Generic Transportation Model)*

Solution of the algebraic equations above with the third order approximation

For illustration purposes, we solve the algebraic equation for the reduce longitudinal equation expand at the bifurcation point and came up with the following solution of the regulating functions:

We expand the equilibrium equations (4.3.22) while substituting equations (4.3.23). We make a Taylor expansion of the states and controls as function of two control inputs.

Computation of the observer gains with poles placement and outputs equations

Observer is computed to asymptotically stabilize the following equation $(A(\mu) + LC(\mu))$:

we have the following observer gain obtained for a particular position of the poles chosen randomly and fast enough compare to the controller poles.

$$L = 10^3 \begin{bmatrix} 0.0239 & 0.0030 & 0.0017 & -0.0461 \\ 0.1952 & -0.0454 & 0.0469 & 3.5091 \end{bmatrix}$$

we then have the observer dynamic that is added to the original state equation above for final implementation where the error dynamic can also be added to the system for a compact structure.

$$\dot{\hat{x}} = A\hat{x} + \varphi(\hat{x}) + L(y - \hat{y})$$

The final Implementation use the following compact structure where the controller uses as input the error and the estimated states:

Compact Structure and Implementation in Simulink/MatLab

Results from Full Structure Implementation

Aircraft Recovery Response with the target point in the safe set

For this specific analysis, we choose to regulate a point in the safe set for which the speed is at 140.239ft/s. the goal of using such model is because it increases the region of attraction and maximize the chances of reaching the target point. Below is the response with respect to that target point.

4.4 conclusion

The controller certainly can be designed theoretically but implementation may be very difficult because we may not have enough control authority to restore the aircraft back to the safe set The

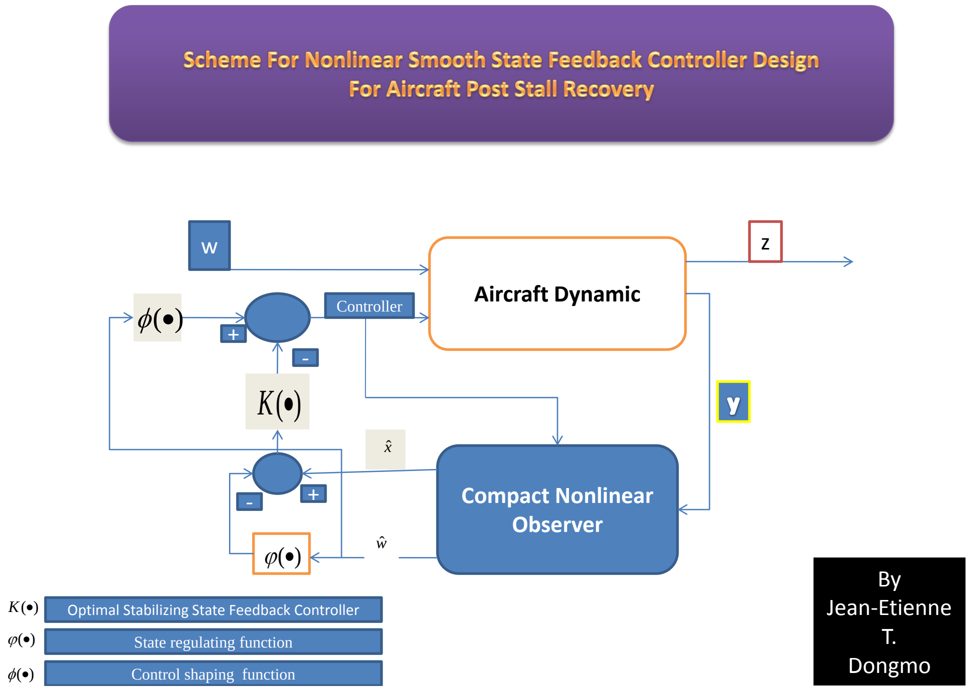
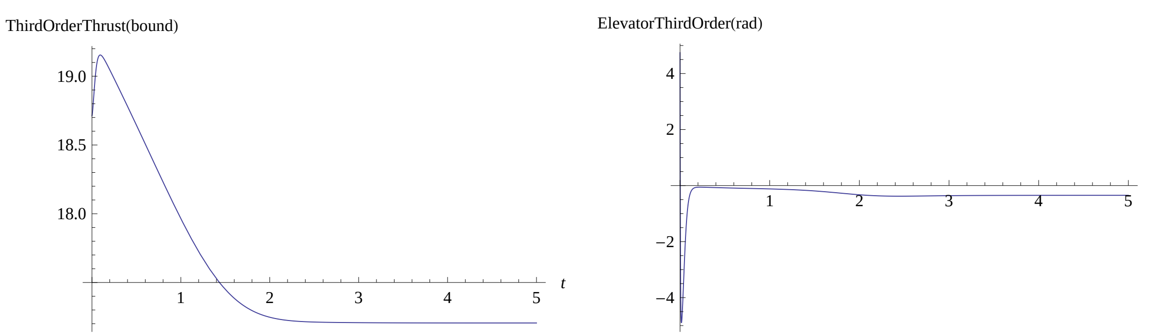


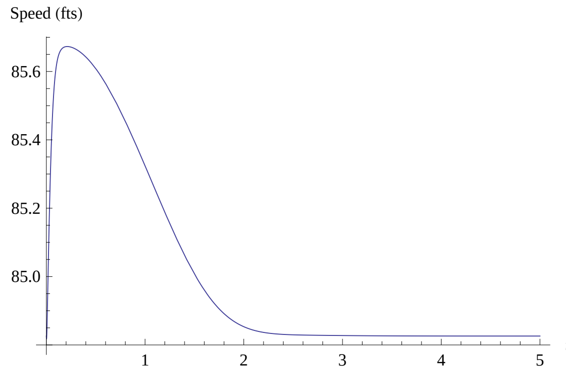
Figure 4.7: nonlinear Smooth Scheme



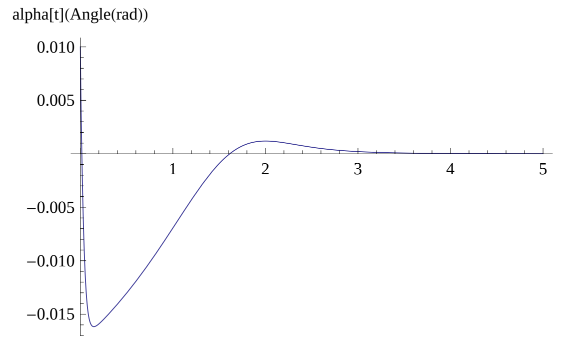
(a) Thrust in the Nonlinear Approximation

(b) Elevator in the Nonlinear Approximation

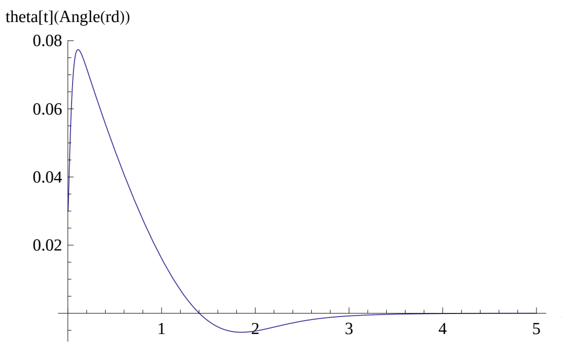
Figure 4.8: Nonlinear smooth feedback regulator control input for third Order Taylor approximation during the recovery process(Controller + Observer) at the bifurcation point



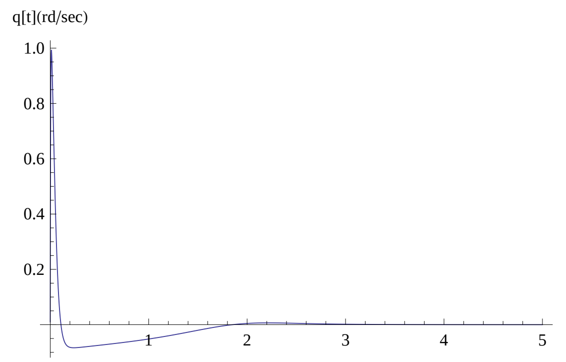
(a) Aircraft speed response of the third order approximation



(b) Aircraft Angle of attack response of the third order approximation

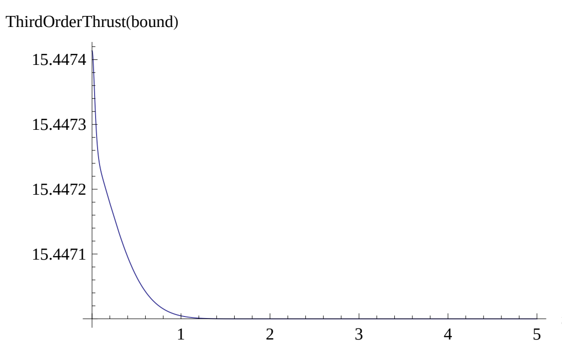


(c) Aircraft Pitch angle response of the third order approximation

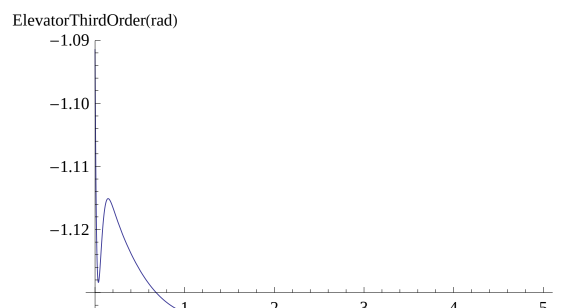


(d) Aircraft Pitch rate response of the third order approximation

Figure 4.9: Aircraft Longitudinal Response with a third Order Taylor Approximation during the recovery process (Controller + Observer) at the Bifurcation Point

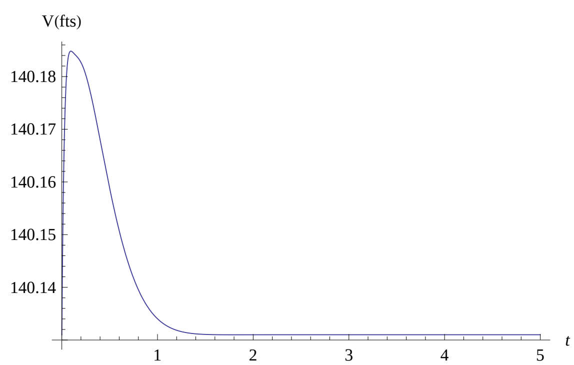


(a) Thrust in the Nonlinear Approximation

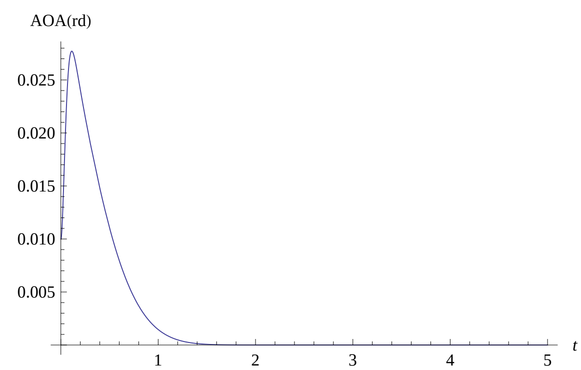


(b) Elevator in the Nonlinear Approximation

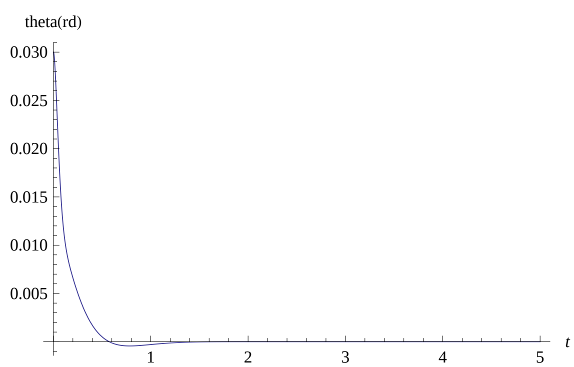
Figure 4.10: Nonlinear smooth feedback control input for third Order Taylor approximation during the recovery process(Controller + Observer) Target Point in the safe set at 140.139fts



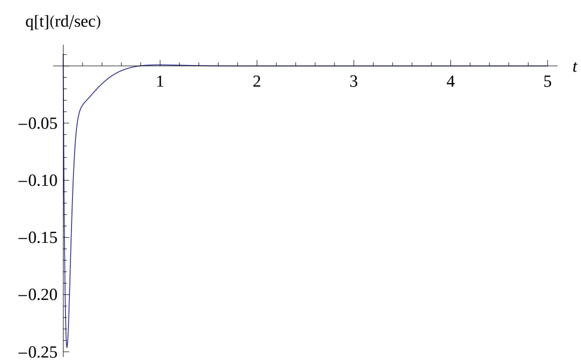
(a) Aircraft speed response of the third order approximation



(b) Aircraft Angle of attack response of the third order approximation



(c) Aircraft Pitch angle response of the third order approximation



(d) Aircraft Pitch rate response of the third order approximation

Figure 4.11: Aircraft Longitudinal Response with a third Order Taylor Approximation during the recovery process (Controller + Observer) target Point in the safe set.

example is at the bifurcation point where the elevator does not have enough room for few seconds but a delay in the design can resolve the issue. As an alternative, we provided the design of switching controllers using high order sliding mode controller. Switching controllers present a package of benefits over the nonlinear smooth controllers such as robustness to parameter variation and finite time stabilization to the regular sliding manifold or zeroing manifold. An other interesting point to make here is the fact that we assume that all states are measurable which is not the case because in flight control system, we always need an observer but the separation principle allow us to make separate design. The next full implementation would require the design of the observer as was done in the full recovery control law of the NASA Aircraft Generic Transportation Model.

In reality,the nonlinear terms in the controller performs a good job in quenching the limit cycle that was in generation process allowing stabilization and maneuverability to safe domain. Such controllers appears to be very interesting in the recovery process where an aircraft can be steer from an unstable mode to a safe environment where the pilot can gain control of the vehicle. Moreover those controllers appear to be very important in flight maneuvers because they increase the region of attraction around each equilibrium point facilitating the recovery process. Also with such technique, regulators can be designed at any trim point including bifurcation points. The most important questions to be answered are: do we actually have enough control authority to restore the aircraft back in the maneuverable safe domain? How fast does the recovery process take place? Is the structure integrity of the airplane maintained? such questions would be answered in the order and the possibility of using switching controllers as an alternative is derived through feedback linearization in the next section.Others questions would be answered during analysis and implementation of the prescribe controller.The algorithm uses is attached in the appendix of the thesis and should be extended to a six degree of freedom model.

5. Aircraft Stall Recovery Using Switching Controllers

The motivation of this section comes from some well known facts among which the non possibility of using a smooth controllers [57] near the critical points especially in flight domains where maneuverability near those points require long standing training . Although the preceding chapter derives nonlinear smooth controllers for stabilization and regulation near critical flight regimes but the probability of not having enough control authority is of concern. In this section of the chapter, we approach the recovery process with a different strategy where feedback linearization [58, 59, 60, 20] is used as a tool to derive switching recovery control law. The technique is attractive in the aerospace industry because it uses state feedback and coordinates transformation to cancel nonlinearities in the nonlinear system and generate a linear system that can be managed. Coordinates transformation is usually the difficult task but a summary of the construction can be found in Isidori et al [58].

The attraction and elegance of this technique comes from its simplicity and the fact that interesting linear control system tools can be used interactively. Important concepts are also introduced such as the *relative degree* which play a major role in analyzing the input/output structure of the system. It's true that the output of the system can usually be structured in such away that the system is feedback input - output linearizable but in most situation in which exact full state linearization is not possible. In such cases the notion of *internal dynamic* leverages the uncontrollable and unobservable mode is of particular importance. Stabilization of the internal dynamic is essential for regulation and tracking of flight control systems and comes as a limitation of the used of other known classical techniques.

This technique plays a significant role in understanding others advanced control design techniques such as variable structure control which is going to be the core of this chapter. In formulating the flight recovery process from a post stall regime to the safe mode, we would proceed by deriving dynamic output feedback and then build switching controllers from the derivation of feedback linearization.

In flight control system design,several authors have successively used the different classical techniques in a whole variety of issues ranging from various aircraft models to different control problems. Stengel et al [50] design control systems for highly maneuverable aircraft where dynamical modes can be changed based on the flight conditions but they do not focus on the stabilization of the internal dynamics. Singh et al [103] restructure the outputs(roll rates,sideslip and AOA) such that they are independently controlled by the different deflection surfaces. In Xinhua et al [63],output tracking controllers are designed with an emphasis on the stabilization of the internal dynamics.

Most important control problems are tasks under large uncertainties especially in flight control because of aerodynamic coefficients not precisely known, failure, environmental disturbances,etc... In most circumstances,the best approach in dealing with uncertainties of all kinds is with *brutal forces* [64]. In flight control system, failures can be a catastrophic disturbance and in dealing with such events, variable structure control is popular technique commonly used because of its robustness to parameter variations and its intuitive approach in performing rapid action [65, 67]. The problem that we face here in our context of post stall dynamical flight is the expectation of an uncontrolled maneuvers because the flight vehicle is performing a set of unpredictable actions. The goal here is to minimize altitude lost and also at the same time to restore the flight vehicle into the safe and controllable mode. Dealing with those particular tasks, we formulated the post stall recovery and use high order sliding mode control to derive a controller that will restore the flight vehicle into its maneuverable regime. The problem is tackled in two steps: first, approach the problem using feedback linearization and second address the computation of switching controllers by choosing the initial sliding surfaces as the error equations used in deriving feedback linearizing controllers.

In our formulation of the recovery process, we are certainly concern with the stability of the internal dynamic but we also concerned with the structural integrity of the vehicle in going from a post stall mode to a safe mode. With that in mind, we used Linear Parametric Varying (LPV) model at the stall point to address all the issues cited above and design the controller for the recovery process. Before we emphasize those issues in details, let's outline the steps for the control law design process.The chapter is organized as follow:

first a few definitions in the variable structure context, second proceed by deriving dynamic output feedback and then close the chapter with an extension of dynamic feedback as switching controllers.Either approach appears to be a contribution to the thesis and observation of the stall/spin prediction is outlined at the end of the thesis.These techniques are all build into Simulink (Matlab) software for advanced autopilot design. In this particular chapter, we reformulated the post stall recovery process and used variable structure control to derive a high order sliding mode controller which allow us to restore the flight vehicle into the safe set.

5.1 Important Definitions and Theorems

There are unlimited number of theorems and definitions that can be used in the context of approaching the design of switching controllers. In this thesis, just those intrinsically related to high order sliding mode control are highlighted.As a starting point, it is important to notice that high order sliding mode comes as a generalization of the basic concepts of standard sliding mode and acts on high order derivative of the system deviation from the constraints.A set of supertwisting [54] algorithms are derived based of the vector relative degree of the system and are of particular importance because of chattering disappearance addressed.

Definition 5.1.1 (Fillipov's set [93]). A curve $x(t) \subset \Re^n, t \in [t_0, t_1]$ is said to be a solution of the aircraft dynamical equation in the neighborhood of a discontinuous surfaces on $[t_0, t_1]$ if it is absolutely continuous on $[t_0, t_1]$ and for each $t \in [t_0, t_1]$

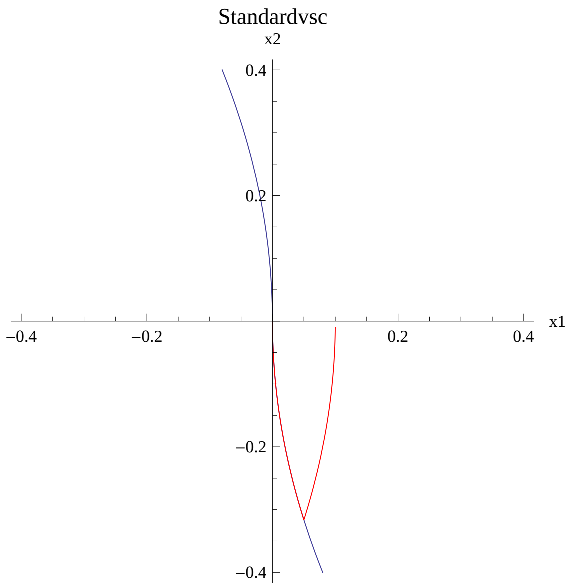
$$\dot{x} \in \tilde{F}(x(t), t) := \bigcap_{\delta > 0} convF(S(\delta, x(t)) - \Lambda(\delta, x(t)), t)$$

Where $S(\delta, x)$ is the open sphere center at x and with radius δ, $\Lambda(\delta, x)$ is the subset of measured zero in $S(\delta, x)$ for which $\dot{x} = F(x, t)$ is not defined and $conv\{F(\cup)\}$ denotes the convex closure of the set of vectors $F(\cup)$

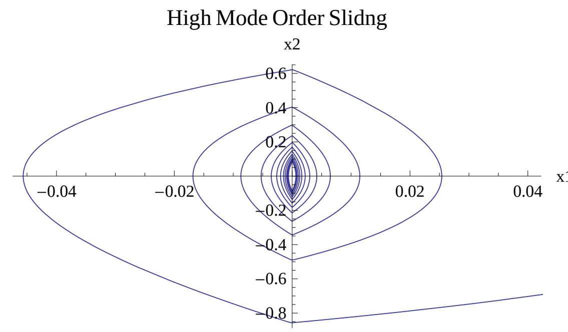
The definition defines the trajectory in the Filippov's with respect to the flow above sense in the vicinity of a sliding surface chosen or designed based on the nature of the problem.

Theorem 5.1.1 (r-sliding mode). *It is said that there exits r-sliding mode of $S(\delta, x)$ in the vicinity of an r-sliding point x if in the vicinity of x, the r-sliding set $S_r(\delta, x)$ is an integral set ie. it consist of a set of Filippov's sense trajectories [94].*

Based on those definitions, the example below illustrate the difference between standard sliding mode and high order sliding mode trajectories before the general process of chosen the regulate outputs for the design of controllers to enforce sliding. Extremely important when approaching the aircraft post stall recovery problem.The figure below shows the simple application of output feedback using high order sliding mode controller and the standard sliding mode control technique.



(a) Standard Sliding Mode Control



(b) High Order Sliding Mode Control

Figure 5.1: Basic Example Comparing High Order Sliding and Standard Sliding

5.2 Recovery Control Law Design Using Feedback Linearization

While addressing the recovery controller design procedure, the structural integrity of the system is maintained by extending the LPV model such that the control rates are designed instead of the physical control applied to the systems. In addressing switching controls, it is always important to master and understand the feedback linearization process because it is first step in the derivation.A twofold problem allow us to use an extended model instead of the actual model: first remove stresses in the model during the recovery and second avoid singularity of the decoupling matrix as it may be the case at the bifurcation point (Stall Trim Point). Moreover, extending the model in a switching control law design, allows chattering which is a major problem in variable structure system to be addressed because chattering take place in the circuit [64]. Robustness and performance of the flight control system design can be easily addressed locally and globally for switching controllers using the outline design procedure:

1. We compute the LPV model and the interested reader can see Kwantny et al [69] and It is also describes in the appendix.

2. Once the LPV model is derived,it fits the affine in control system necessary for feedback Linearization technique.

$$\begin{aligned} \dot{x} &= f(x,\mu) + G(x,\mu)\,u \\ e &= h(x,\mu) \\ x \in R^n,\, u \in R^m, e &\in R^m, \mu \in R^q \end{aligned} \tag{5.2.1}$$

3. We performed an extension of the model by adding integrators in front of the physical controllers (Thrust,elevator,aileron,rudder), the intend being to allow smoothness during the recovery process. The extension procedure is called Dynamic Extension [58, 95].

$$\begin{aligned} \dot{w}_1 &= Th \\ \dot{w}_2 &= dele \\ \dot{w}_3 &= dela \\ \dot{w}_4 &= delr \end{aligned} \tag{5.2.2}$$

   Combining (5.2.1) and (5.2.2),we have the complete affine extended aircraft model with slightly four more states added in the complete affine system then ready for the recovery process.

4. We computed the transformation which allow us to decouple the system into the linearize part and the nonlinear part. Using the compact system (original system plus extension), we developed a sequence of partial linearizing technique using Lie Bracket of the outputs as describe by Kwatny et al [70].

5. Once the Linearize part is obtained,any classical control tools such as the LQR, Pole placement,H_∞ can be used to derived the linear controller and in our case,we used high order sliding mode control technique for finite time stabilization on the zeroing manifold.

6. The final controller is then derived based on the previous linearize design controller.

Application of the algorithm

The complete system after extension is then present as follow:

$$\dot{x} = f(x,\mu) + g(x,\mu)u$$
$$y = h(x,\mu) \tag{5.2.3}$$
$$x \in \mathbf{R}^n, u \in \mathbf{R}^m, \mu \in \mathbf{R}^p$$

We then performed a sequence of *Lie Bracket* derivation of the regulated outputs error$z(x) = y - y_0 = h(x) - y_0$ until at least one of the controllers appears:

$$L_f(z_j(x)) = \langle d(h_j(x,\mu) - y_0), f\rangle = \frac{\partial(h_j(x,\mu) - y_0)}{\partial x} f(x,u,\mu)$$
$$. \tag{5.2.4}$$
$$.$$
$$L_f^k(z_j(x)) = L_f(L_f^{k-1}(z_j(x))) \quad for \quad j = 1, ..., m; \;\; k = 1, ...r_j$$

where r_j is defined to be the *relative degree* associated to the particular output j.

Definition 5.2.1 (**Vector Relative Degree**)**.** Let's $r_j = \inf\{k|L_{g_j}(L_f^{k-1}(h_i(x) - y_0^i)) \neq 0 \;\; for \;\; at \;\; Least \;\; one \;\; j\}$, Then $r_1, ..., r_m$ is defined as the vector relative degree associated to the system(5.2.3)

Using the following transformation $x \to (\xi, z)$ where $\xi \in \mathbf{R}^{n-r} \wedge z \in \mathbf{R}^r$, we have the following decouple partial linearizing system:

$$\dot{\xi} = F(\xi, z)$$
$$\dot{z} = Az + E(\sigma(x) + \rho(x))u$$
$$y = Cz \tag{5.2.5}$$
$$z_k = L_f^k(h_j), \;\; k = 1, ...r_i \;\; and \;\; i = 1, ..., m$$
$$\sigma_i(x) = L_f^r(h_i) \;\; i = 1, ...m$$
$$\rho_{ij}(x) = L_{g_j}(L_f^{r-1}(h_i)), \;\; i, j = 1, ..., m$$

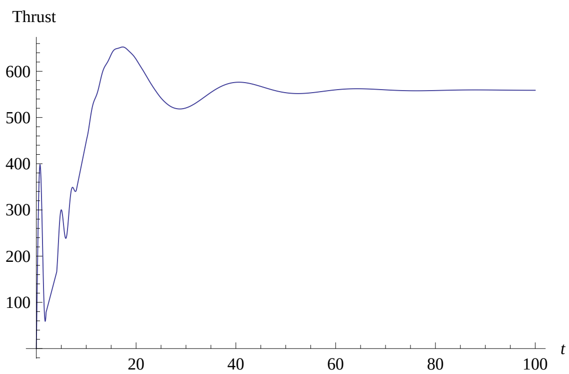
where with the following nonlinear control $u(x) = \rho^{-1}(x)\{v - \alpha(x)\}$, we have the following linear decouple system:

$$
\begin{aligned}
\dot{z} &= Az + Ev \\
y &= Cz \\
A = diag(A_1, ..., A_m) \quad with \quad A_i &= \begin{bmatrix} 0 & I_{r_i - 1} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{r_i \times r_i}
\end{aligned} \tag{5.2.6}
$$

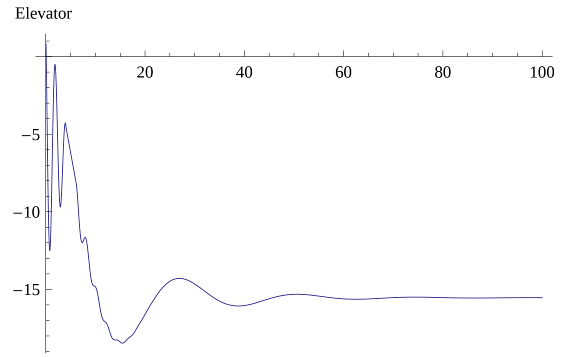
A few questions arose when it comes to design the fictitious controller v of the linearize system 5.2.5. It is true that there exists well known tools that help shape the design of v but in our case, we are used High Order Sliding Mode Controller(HOSMC) to ensure finite time stabilization on the zeroing manifold and to avoid perturbation of the zero dynamics. In this specific context,we have a structurally unstable zero dynamic and its stabilization is critical while restoring the aircraft into the safe mode. Another question is how do we design v to avoid saturation of the actual controller $u(x)$ since they are always bounds? This questions is answered in the next upcoming section.

Example A six degree of freedom aircraft is used where the models is slightly expand at the bifurcation point (LPV model is derived at the bifurcation as shown in [71]) with the assumption that it is in a post stall mode where a number of know facts can be observer as state earlier in chapter four ranging from inertia cross coupling to instability of the zero dynamic. The graph below shows the response of the flight vehicle from a post bifurcation (Post Stall). The model used here is expand and load in the appendix B of the thesis. The control problem here is to restore the flight vehicle to the safe flight mode, by making regulation to a known target point.

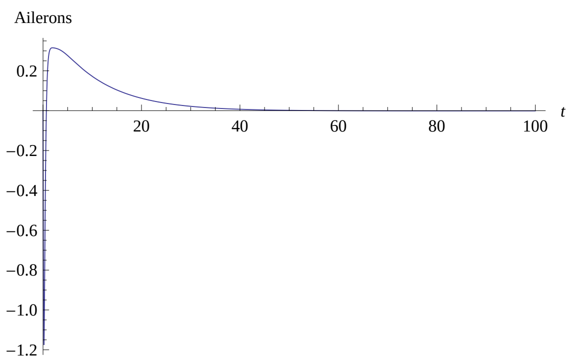
As the plot above indicate, the vehicle fully recover without any problem with the only exception that, the thrust is completely out of the bounds. It appears that we needed the thrust to ensure that we can regain enough lift by increasing the speed once the vehicle is stabilized. Surprisingly the deflection surfaces behave within the normal bounds and mimic the steps that the pilot would normally go through to recovery. The only difference is that in 30 seconds, we fully recover, loosing less altitude. our only concerns are the status of the flight vehicle structure and the bounds on thrust. Previous works [105] attempt to demand additional source of power for better recovery. In our first attempt to resolve the problem, we use switching controllers described below where the bounds on the controls are directly addressed. Then we reformulate the problem using linear matrix inequalities. Section 5.3 will go through the steps in details. Another important observation from



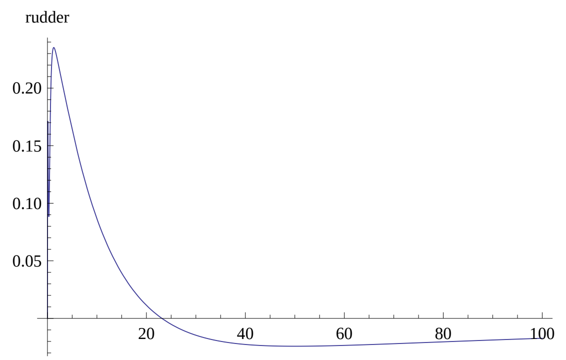
(a) Thrust in Post Stall



(b) Elevator in Post Stall

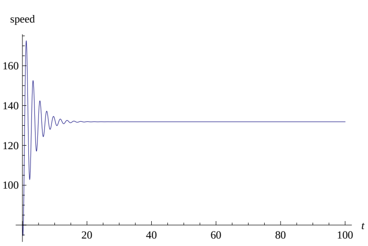


(c) Ailerons in Post Stall

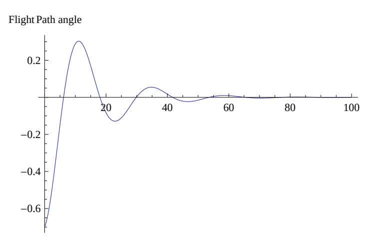


(d) Rudder in Post Stall

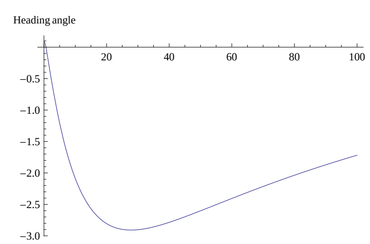
Figure 5.2: Aircraft Deflection surfaces from a post stall mode using Dynamic Output Feedback Controller



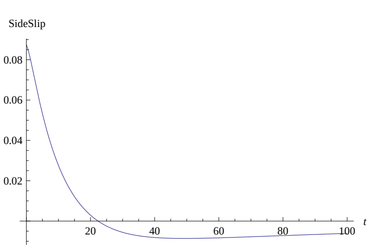
(a) Speed during the recovery process



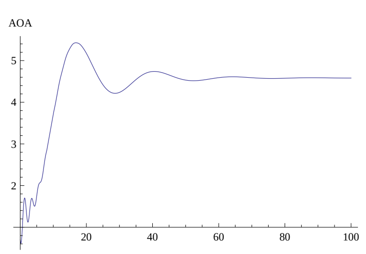
(b) Flight Path during the recovery process



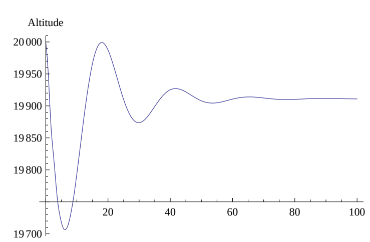
(c) Heading Angle during the recovery process



(d) Bank Angle during the recovery process



(e) Angle of Attack Angle during the recovery process



(f) Altitude Drop during the recovery process

Figure 5.3: Aircraft response from a post stall mode using Output Feedback Controller

this recovery process is that as the angle of attack increases the further we drop in altitude. The question now is when should we start the recovery to minimize altitude drop?

5.3 Design of Linearize Controller Using High Order Sliding Mode Controller

The goal of this section is to improve the recovery strategy from a post-stall mode to the safe mode. Improve the aircraft recovery strategy could mean many key things minimization of the lost of altitude, performed a quick recovery process, avoid overstress of the flight vehicle and finally account for the passenger comfort. Satisfaction of all these requirements at once can be difficult, but at this stage that we have confidence that the process would work because the previous technique show satisfactory results, we use a high order sliding mode controller because of its robustness to parameter variation and sliding manifold finite time reachability and stabilization. A contribution in this particular section relies on the design of aircraft recovery switching controller while maximizing the region of attraction around the stall equilibrium point.

High Order Sliding Mode Controller (HOSMC) comes as an extension of the standard and well known Sliding Mode Controllers (SMC) technique [71, 73]. HOSM controllers have the key attribute of stabilizing to an rth order sliding manifold in finite time as will be seen in below in this section. They also add robustness with respect to disturbances [74]. In the case of aircraft recovery, we require a robust, fast response strategy that maintains the structural integrity of the aircraft. We also need to minimize altitude loss. In order to design the HOSM stabilizing controller, we have to define an rth order sliding. The design of HOSM controllers is performed in two majors steps. First is the choice of sliding surface or the design of sliding surface [78] and second is the design of the reaching controller that brings aircraft to the sliding surface. Designing the sliding surface in our case is not too difficult because we know where we want to be in the state space or the path that we can follow to reach the safe set or the safe mode in final time. Instead of designing optimal sliding surface as it is the case with others class of systems, we choose our initial sliding surfaces as error equations which in the aircraft context, the following can be chosen as initial sliding surfaces and the number of components should be less or equal to the control inputs $Thrust$, $dele$, $dela, delr$

$$S_{init} = \{V - V_0, \gamma - \gamma_0, \mu - \mu_0, \psi - \psi_0\} \tag{5.3.1}$$

Where V is the speed, γ is the flight path, μ is the bank angle,ψ is the heading angle while the following are the regulating target values $\{V_0, \gamma_0, \mu_0, \psi_0\}$ belong to the safe set. Once the system is decoupled from 5.2.3 to 5.2.5 and once the system is converted in 5.2.6, then optimal sliding surface can be design as in [77; 68] as opposed to the choice we made in my design. Further investigation in the same work recovery idea would be investigated as future works in comparison with they work already done. Following the derivation from Feedback Linearization(FL), the final set of r - th order sliding surface would be $\{S_i^j, i = 1, ...p;\ j = 1, ..., r_i - 1\}$ with $\{r_i\}_{i=1}^p$ vector relative as defined above.

In fact the final sets of sliding surfaces are the set of initial regulating outputs and their high order derivatives. Our goal is to force the initial surface and their high order derivative to zero at the same time during the recovery process. The final components of the sliding surfaces are: $\{z_i, ..., z_{r_i-1}\}$ and those components are zero in finite time is exactly the same as making the linear combination equal or a nonlinear combination equal to zero as it is the case in high order sliding mode controllers. In our context, we would follow Kwatny et al [70] but would choose our sliding as a nonlinear combination of the initial and high order derivatives up to $r_i - 1$ where r_i is the relative degree associated to the output. In our case, the sliding surfaces would be a combination of these elements

$$\begin{aligned}&\{Sign[z_i^j]Abs[z_i^j]^{\nu_i}\}\ \ \forall i \in \{1, ...n\}\ ,\ \forall j \in \{1, ...,r_i - 1\}\\&with\ \nu_i \in (0,\, 1)\end{aligned} \tag{5.3.2}$$

Once the sliding surfaces are chosen, our goal is to compute the controller that will steer the aircraft back to the safe set in finite time with finite time stabilization and reachability of the error zeroing manifold which is an improvement of the approach performed by kwatny et al [70] where asymptotic stabilization was allow. This step is analyzed using Lyapunov theorem. Before we proceed, a set of definition have to be made:

Definition 5.3.1 (**Finite Time Reachability of the Sliding Manifold [83]**)**.** The Origin or the flight trim point x_0 is finite time reachable and stabilizable of the dynamical equation describing the aircraft if there exists a neighborhood of the trim point$\aleph \subset \Im$; $0 \in \Im$ and a function $T : \aleph \to (0, \infty)$, called the *settling time* for all x in the neighborhood of the trim condition such that the following conditions hold:

1. Finite time convergence: $\forall x \in \aleph \backslash \{0\}$ the flow function $\Psi(t, x)$ is defined for all $t \in [0\ \ T(x)]$ and $\Psi(t, x) \to 0\ as\ t \to T(x)$ (Here we see that the Exponential Map of ProPac Toolbox becomes extremely important in dealing with flow function in hybrid Systems).

2. Lyapunov stability: For every open set $0 \in U_\varepsilon \subset \aleph$, there exists an open set $U_\delta \backslash \{0\}$ such that

$$\Psi(t,x) \in U_\varepsilon \ \ \forall x \in V(0)$$

Definition 5.3.2 ($\mathbf{r}_(th) Order Sliding Manifold$[68]). Consider the set defined below: $S^r = \{x : Z_i(x) = ...Z_i^{r_i - 1} = 0,\ for\, i = 1, ..., m\}$ where the time derivative of the zeroing outputs is equal to zero up to term and is non empty, then the set is called the $r_t h$ order sliding mode set and the motion in that set is called the $r_t h$ order sliding mode controller with respect to the initial zeroing output variables.

One of the most fundamental reasons for using high order sliding mode controllers is that it is well known that sliding mode controllers have difficulties stabilizing systems with relative degree greater or equal two [75] with unstable zero dynamics. Moreover, high order sliding mode controller appears to be robust with finite time stabilization on the sliding manifold. Although Dongmo et al [76] were able to achieve speed regulation with exponential stabilization of variable structure control with partial linearizable dynamics using the approach of Kwatny and Kim [70]. The main reason being the chattering process that were taken place across the sliding manifold as opposed to high order sliding modes were within the circuit [78].

The transform extended model (5.2.1) and (5.2.2) can be decoupled into the following Brunowski form and the controllable part of the Brunowski form represents the chain of integrators that need to be stabilized in *finite time*. We had chosen high order sliding mode to stabilize the chain of integrators because of its robustness and its unstructured internal dynamics [82]. The following theorem provides guidelines for peaking the coefficient for stabilization and the reaching time to the sliding manifold can be computed.

Theorem 5.3.1 (Integrator Stabilization). *Let's the positive constant $c_{1,i}, ..., c_{r_i,i}$ be such that the following define polynomial$s^{r_i} + c_{r_i,i}s^{r_i - 1} + ... + c_{1,i}$ is Hurwitz. There is an $\varepsilon \in (0,\, 1)$ such that for every $\gamma_i \in (1 - \varepsilon_i,\, 1)$ the set of integrators is stabilized in finite under the following feedback law:*

$$v_i(z_i) = -c_{1,i} Sign(z_{1,i})|z_{1,i}|^{\gamma_i} - .... - c_{1,i} Sign(z_{r_i,i})|z_{r_i,i}|^{\gamma_{r_i,i}} \qquad (5.3.3)$$

Where $\gamma_{1,i}, ..., \gamma_{r_i,i}$ satisfied the following relationship

$$\gamma_{j-1,i} = \frac{\gamma_{j,i}\gamma_{j+1,i}}{2\gamma_{j+1,i} - \gamma_{j,i}},\ j = \{2, ..., r_i\} with \;\; \gamma_{r_i-1,i} = 1\, and\, \gamma_{r_i,i} = \gamma_i \tag{5.3.4}$$

The proof of the theorem can be found in [83].

Theorem 5.3.2 (83)**.** *Suppose there exists a continuously differentiable function $V : D \rightarrow \Re$, a real number $k > 0$ and $\nu > 0$ where D is a neighborhood of the trim point such that the function $V(x)$ is positive definite on $U \subset D$ and $\dot{V}(x) + kV(x)^{\nu}$ is negative definite on U, then the trim point is reachable in finite time and is also stable.*

Once the sliding surfaces are designed and the system decoupled as shown above, the linearize part of the system is a set of block integrators which are the sliding surfaces which should be forced to zero in finite. And the second step of the designed is to derive the controller that would allow reach ability of the sliding manifold in finite. In doing so, we used the following theorem from Kwatny et al [70] to ensure that the final designed controller is stable and reach the high order sliding manifolds are finite time reachable. Before we used the theorem for controller design, we have to make the following observation based on the following theorem for finite time stability.

Lemma 5.3.3. *For all sliding manifold $z \in \mathbf{R}^r$, we can always have validated the following statement:*

$$Sign(z)|z|^{\nu} \leqslant z \;\; or \;\; Sign(z)|z|^{\nu} \;\; \forall \nu \in (0, 1) \tag{5.3.5}$$

We know that with finite time stable theorem that finite time imply asymptotic stability but the converse is not true. With $z \in \Re^n$ as a vector of component that defines the sliding manifold we can always manage to design controllers for asymptotic stabilization. Because with the following component $Sign[z]\,|z|^{\eta}$ we can have finite stabilization then

$$\forall z \in \Re^n \;\exists \eta \in (0\,,\,1)\; s.t\; sign(z)|z|^{\eta} < z \Rightarrow \eta < \frac{Log[z/sign[z]]}{Log[|z|]} \tag{5.3.6}$$

In Kwatny et al [70] the following following theorem is used to derive the controller that would ensure stabilization and regulation across the chosen sliding surface. In our context we make used of that theorem and the lemma above to show that we have a high order stabilization and regulation controller with the best possible region of attraction which is an important tools in analyzing nonlinear control system. The theorem also ensure that we can always place pole on certain region of the state space to obtain the require regulation with robust stabilization.

Theorem 5.3.4 (Lyapunov [93]). *Choosing $S = KZ$ where Z has components $sign[z_i]|z_i|^{\nu_i}$ with $i = 1, ...n_j$ and $j = 1, ...m$, we can always find a Quadratic Lyapunov Function $V = S^T QS$ such that the $L_F V(Z) < 0$ With Q a positive definite matrix. A high order sliding submanifold would exists in finite time in a region of the state space such that the time rates of change of the quadratic Lyapunov function is negative using the predesigned high order sliding mode controller.*

Form [70], choosing the controller as follow: $u_i = u_{i\max} sign[\rho(x)^T QKZ]$ always ensure asymptotic stability while reaching the sliding manifold. Because of the lemma above, we see that we can always find the controller such that we have finite time reachability and stability with

$$V(z) = (KZ)^T Q(KZ) \Rightarrow \dot{V}(z) < 0 \tag{5.3.7}$$

With the right choice of controller as shown above. Replacing Z by a linear combination of $sign[z_i]|z_i|^{\nu_i}$, we know that

$$\sum_{j=1}^{n_i-1} k_j sign(z_j)|z_j|^{\nu_j} \leq \sum_{j=1}^{n_i-1} k_j z_j \tag{5.3.8}$$

These inequalities hold from the above theorem since $k_j > 0$

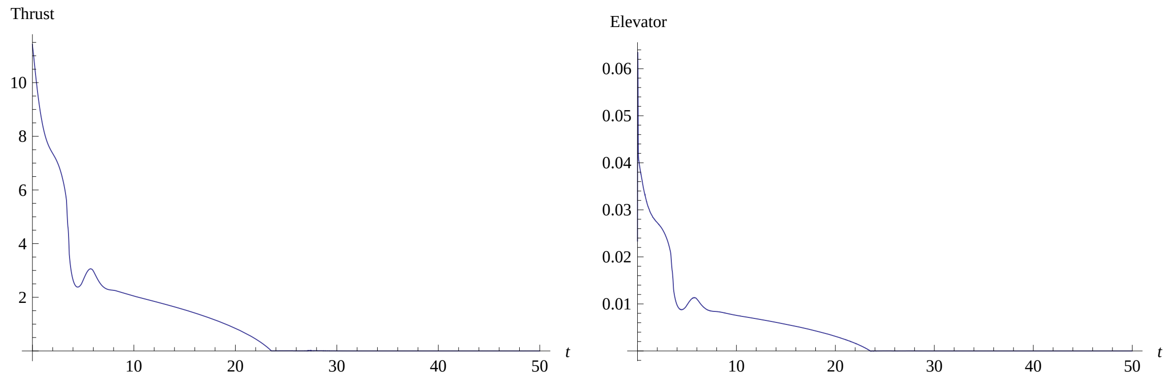
$$\dot{V}(z) = 2[KAZ + \alpha]^T QKZ + 2u^T \rho^T QKZ \tag{5.3.9}$$

We see that Z appears almost linearly in the equation (5.3.9) and using inequality (5.3.7), we have

$$\dot{V}(\sum_{j=1}^{n_i-1} sign(z_j)|z_j|^{\nu_j}) \leq \dot{V}(\sum_{j=1}^{n_i-1} z_j) < 0 \tag{5.3.10}$$

With the following controller:

$$\begin{aligned} &u_j = U_{j\max} sign(\rho^T QKZ) \\ &U_{j\min} \leq u_j \leq U_{j\max} \ \ and \ \ \forall j \in \{1, ...m\},\ Z_j = \sum_{j=1}^{n_i-1} k_j sign(z_j)|z_j|^{\nu_j} \end{aligned} \tag{5.3.11}$$



(a) Internal Thrust in Post Stall (b) Internal Elevator in Post Stall

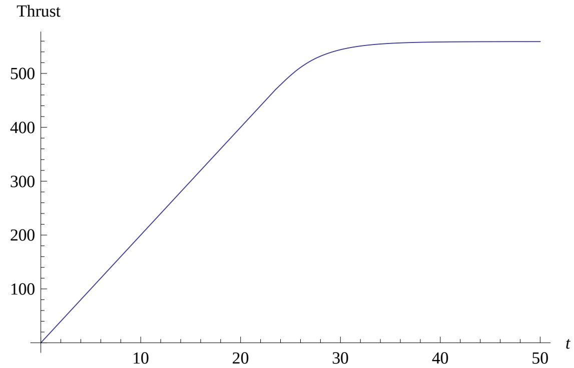
Figure 5.4: Internal Value Deflection surfaces from a post stall mode using High Order Sliding Mode Controller

The test of the controller design above is performed on the NASA Aircraft Generic Transportation Model in a post - stall recovery and it seems to be performing well with the exception that the controllers are out of the bounds and the problem is addressed below. Results are shown below. In this specific recovery we are allow to have the aileron and rudder idle, then use a combination of thrust and elevator to perform the recovery process.

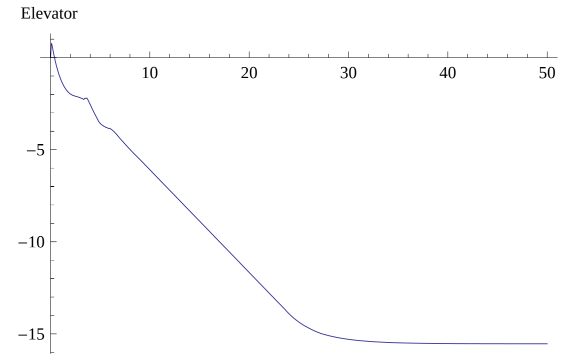
Simulation Results using HOSMC

In This section as describe above, we chose the bounds on the controller to be reasonable enough as the code shows in the appendix B while increasing the region of attraction with picking $Q = DiagonalMatrix[0.0001, 10, 10, 10]$

The problem that we encounter while resolving the recovery process is that the controllers are always out of the normal bounds. Either using the *High Order Sliding Mode Controller(HOSMC)* or the standard feedback linearization case the thrust appears to out of the normal bounds. The realization here is that we can recover quicker with switching controllers than we normally should with the standard feedback linearization controller and we also manage to recover faster with switching controllers. Either technique would be better if they can manage to maintain the structural integrity of the plane while performing the recovery. That assessment would have to be performed nicely by integrating the load factor into the equation to manage its structure. An interesting point to make

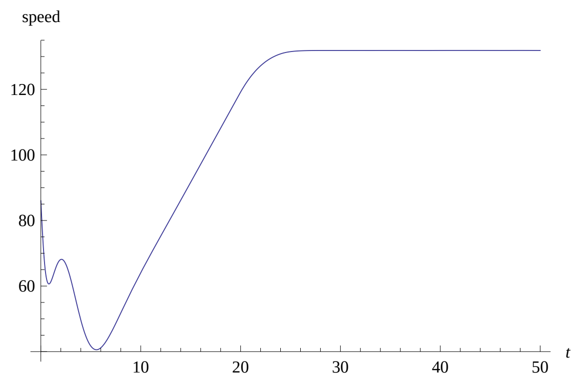


(a) Thrust in Post Stall

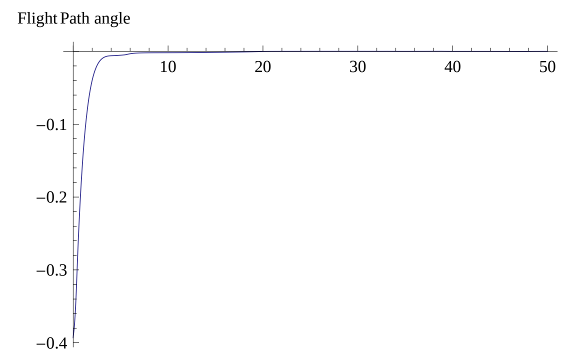


(b) Elevator in Post Stall

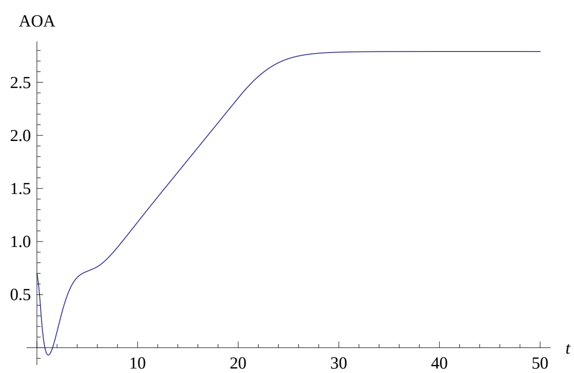
Figure 5.5: Aircraft Deflection surfaces from a post stall mode using High Order Sliding Mode Controller



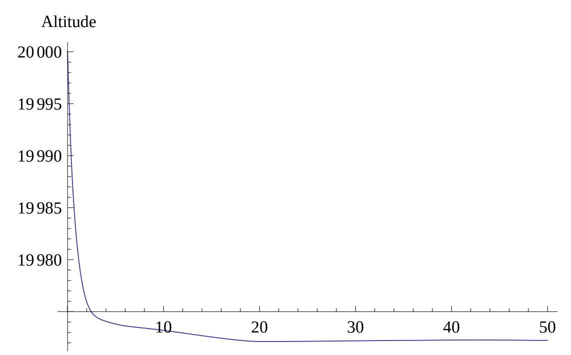
(a) Speed during the recovery process



(b) Flight Path during the recovery process



(c) Angle of Attack Angle during the recovery process



(d) Altitude Drop during the recovery process

Figure 5.6: Aircraft response from a post stall mode using High Order Sliding Mode Controller

here is that we found that reducing the bound on the controller delay the full recovery process and opening the bounds makes the recovery quicker with minimization of altitude drop.

We reformulated the problem using Linear Matrix Inequalities and use linear programming to compute the bounds on the fictitious controllers and on the gains that appears on the fictitious controllers with the assumption that we know the bounds on the physical controllers. The next section explained the process and gives the preliminary results

5.4 Strategy for Controller Saturation Avoidance

In the design above, few problems are encountered amongst the most important being the thrust and or actuators torque out of the normal range for effective recovery. Control inputs saturation is been by many researchers ranging from using antiwindup strategies [84, 85], the application of Linear Matrix Inequalities [86] to the use of Gain scheduling technique [88]. The problem seems to be attraction because important control problems are always subject to constraints. Especially in the flight control community where the pilot always have a predefined flight envelope and must remains inside the flight envelope for all future. Flight maneuver is a challenging task for the pilot because of several operating regimes and within regimes (mode),different control efforts must be applied. From a variety of control techniques use sofar, although successful they have advantage and disadvantage. In our context, we used Linear Programming (LP) to address the saturation problem because of the tremendous advances in computer processors and the complexity of modern system and its necessity to react faster.

Linear Programming is among the oldest technique of optimization which is regaining interest because of the computer era and important advances in power electronics.Important control problems if convert into linear systems can be solved easily and the solution may be simple as possible. In our case, after input/ouput linearization, the actual control after derivation is fictitious controller dependent as shown in the following formula:

$$\begin{aligned} &v = \alpha(z, \mu, \xi) + \beta(x, \mu, \xi)u \\ &u_{\min} \leq u \leq u_{\max} \\ &x_{\min} \leq x \leq x_{\max} \end{aligned} \tag{5.4.1}$$

The fictitious controller v has to be chosen within the reasonable bounds to avoid saturation of the actual controller. In our approach, we make use of the actual bounds on the physical controller to derive the allowable range of gains for the fictitious controllers.

Using the function minimizes and maximize and the constraint on the states and also on the actual controllers, we determine $v_{\min}$, $v_{\max}$ by posing the problem as follow:

$$v_{\min} = \underset{u_{\min} \leq u \leq u_{\max}\ \overline{x}_{\min} \leq x \leq x_{\max}}{Minimize\{}\ \alpha(z,\mu,\xi) + \beta(z,\mu,\xi)u\ \} \tag{5.4.2}$$

$$v_{\max} = \underset{u_{\min} \leq u \leq u_{\max}\ \overline{x}_{\min} \leq x \leq x_{\max}}{Maximize\{}\ \alpha(z,\mu,\xi) + \beta(z,\mu,\xi)u\ \} \tag{5.4.3}$$

with $\{z, \xi\} = \Phi(x)$ from the transformation above to normal form

Once the bounds on the fictitious controllers are known, and it is true that the bounds fictitious bounds are over estimate and one of the reason is that we want to perform full control action in order to restore the flight vehicle into the normal operation mode. It is well known [103] that in order to perform an optimal operation, you have to operate on the boundary of you controllers which is why variable structure controllers are so attractive. Then we compute the bounds on the gains for the obtained controller using any of the linear control technique such as the pole placement case and the high order sliding mode controller as is the case here. Because we were certain that the gains are always positive and they appear linearly in the equation of the gain as it is the case here for a second or a third order integrator, We maximized the sum instead of maximize each component of the gain. As illustration purposes, this is the strategy

Assume that

$$v = -k_1 Sign(z1)|z1|^{\nu_1} - k_2 Sign(z2)|z2|^{\nu_2} \tag{5.4.4}$$

For a second order integrator, then the bounds on the controller are computed using:

$$\begin{aligned} &Maximize\{k_1 + k_2\} \\ &\quad subject\ to \\ &\quad v_{\min} \leq v \leq v_{\max} \\ &\quad x_{\min} \leq x \leq x_{\max} \\ &\quad u_{\min} \leq u \leq u_{\max} \end{aligned} \tag{5.4.5}$$

For a third order integrator the same procedure can be carried and an optimal control case can be used to check the validity of the results. As oppose to previous results, we allowing here saturation of the controller because we want full control authority to regain control the flying of vehicle and also manage steer it back to the maneuverable domain.Below we have the results for the bounds on the fictitious controller and the gains for both high order sliding mode and the dynamic feedback case. The following theorem is then build from previous computation or analogy

Theorem 5.4.1 (**Bounds on the control magnitude**). *If $v(t) \subset V$, ei... each $v_i(t)$ satisfies*

$$V_i^L \leq v(t) \leq V_i^U \tag{5.4.6}$$

*with V_i^L and V_i^U given above by (**??**) and (5.4.3), then*
$u(t) \in U(U\, with\, knownBounds)$

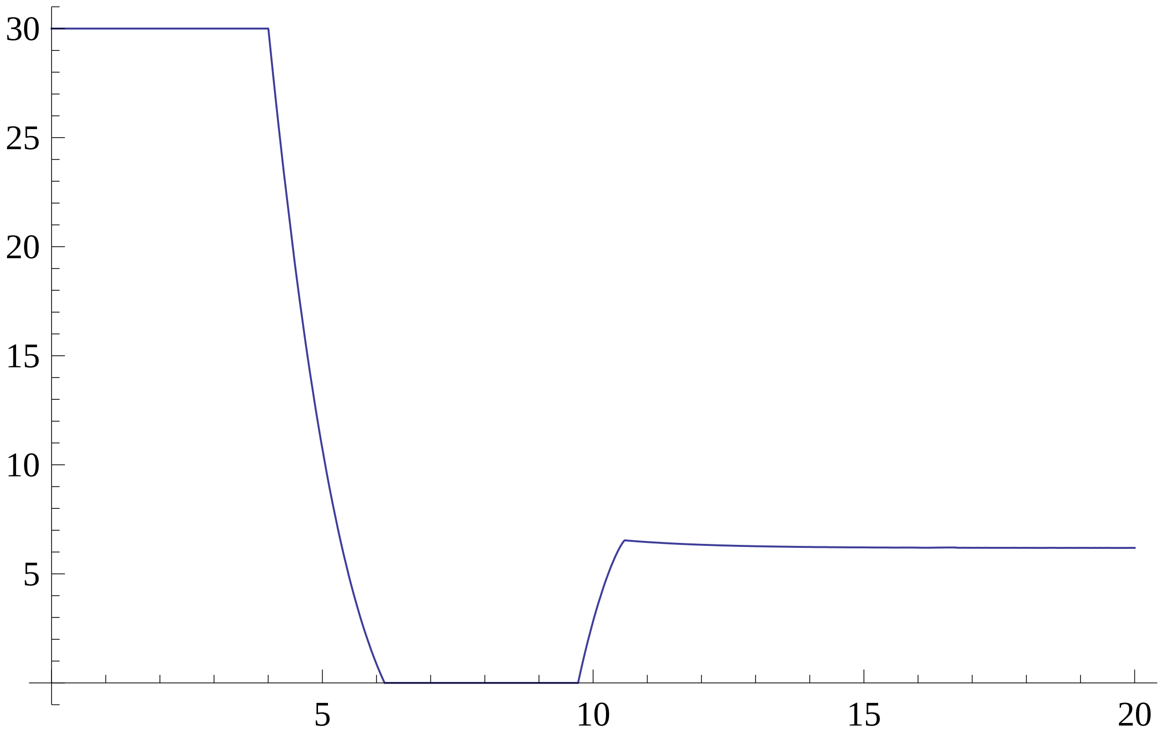
Simulation is then perform to find the optimal gains for reasonable performance. Based on the description of the model that we laid out in the appendix, we have the results the table for a six degree of freedom at the bifurcation point and also and illustration using a nonlinear phugoid mode example.

Example: ***NASA Aircraft Generic Transportation Model: reduced nonlinear phugoid model*** In this particular context we are using a reduce model of the longitudinal model describe in chapter three during prevention analysis to also illustrate the recovery process with controller within the bounds.Here saturation is allow for full recovery.

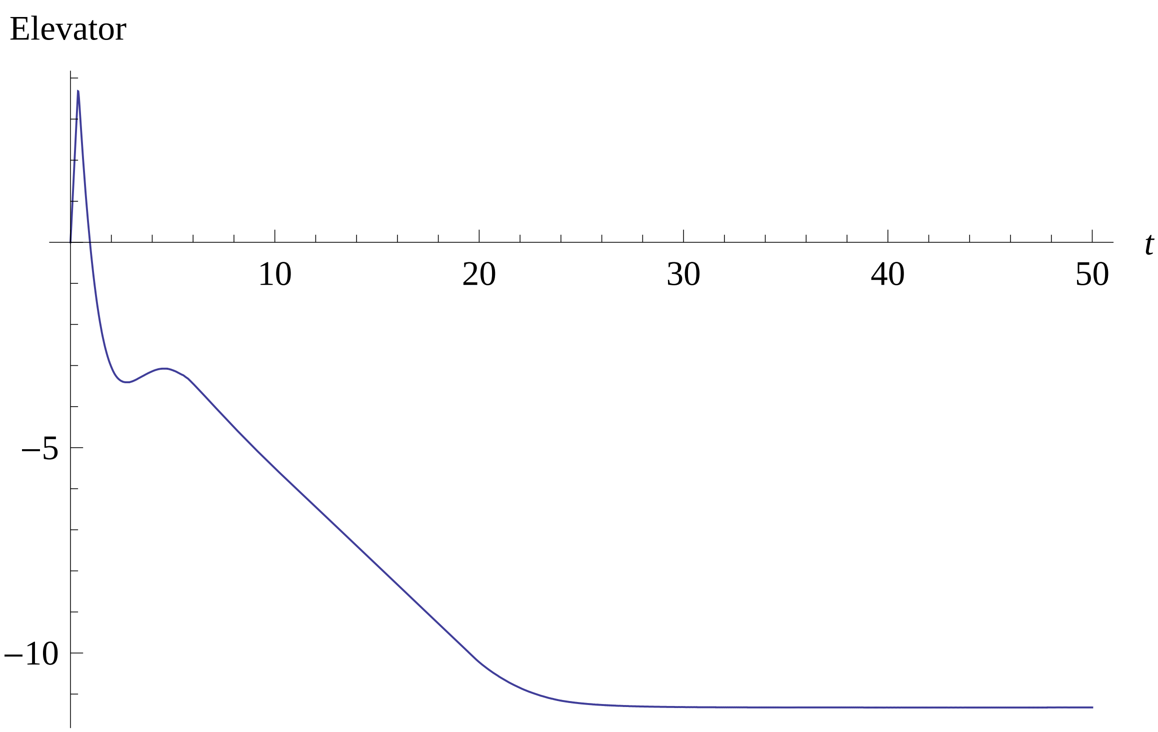
Example:***NASA Aircraft Generic Transportation Model LPV Model (Extended Model at the Bifurcation)***

We used exactly the same model as above and we just manage to control the bound on the controller as require. Then we simulate the system to reduce the level of the controller effort. At this point we made interesting progress by reducing the level of thrust. Adding an additional moment from displacing the thrust axes from the center of gravity play a significant role of adding additional speed that we need to generate enough lift and maintain the aircraft into its normal maneuverable mode.

The following table of numerical data change with the changed of the relative degree, but in this specific case we have (2, 2, 3, and 2) as the vector relative degree associated with the six degree of freedom model. The numbers are associated with the high order sliding mode controllers.

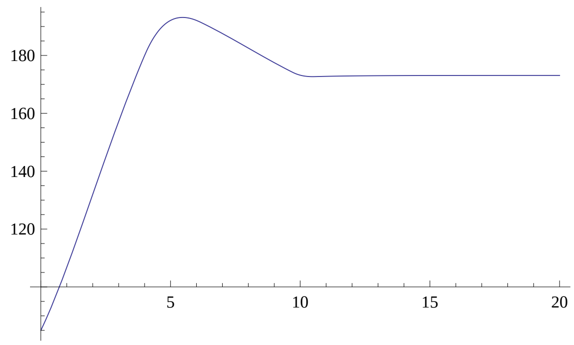


(a) Thrust in Post Stall

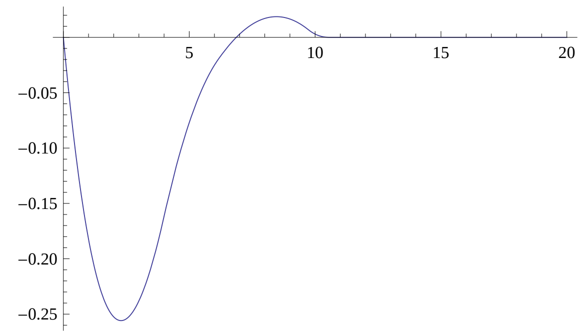


(b) Elevator in Post Stall

Figure 5.7: Aircraft Deflection surfaces from a post stall mode using Improve High Order Sliding Mode Controller

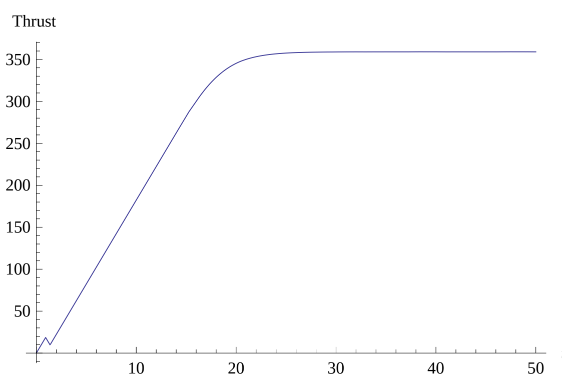


(a) Speed during the recovery process

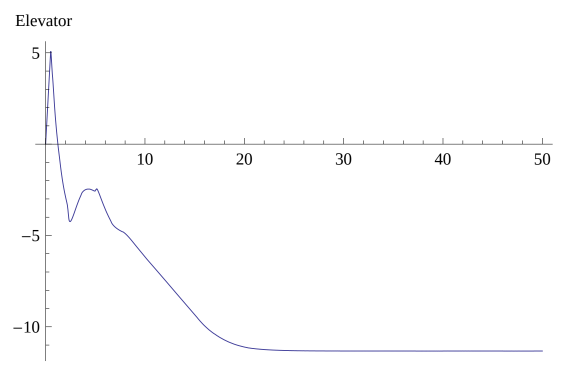


(b) Flight Path during the recovery process

Figure 5.8: Reduce Aircraft model response from a post-stall mode using High Order Sliding Mode Controller

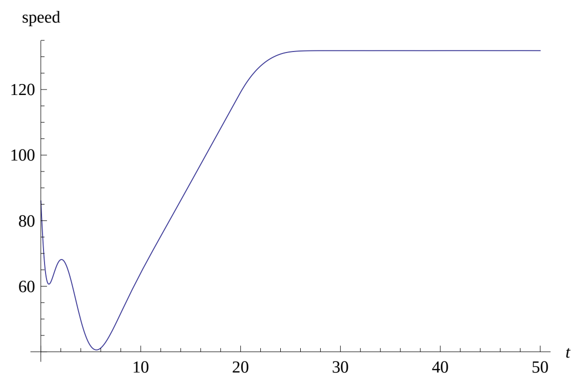


(a) Thrust in Post Stall

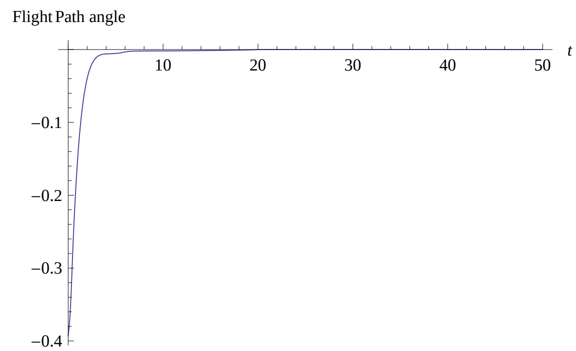


(b) Elevator in Post Stall

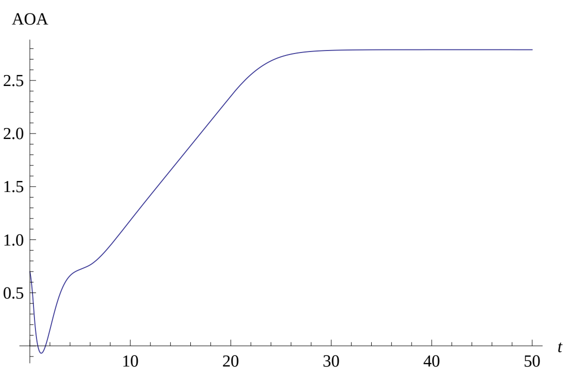
Figure 5.9: Aircraft Deflection surfaces from a post stall mode using Improve High Order Sliding Mode Controller



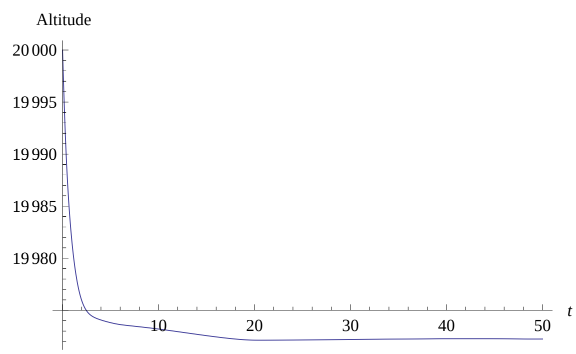
(a) Speed during the recovery process



(b) Flight Path during the recovery process



(c) Angle of Attack Angle during the recovery process



(d) Altitude Drop during the recovery process

Figure 5.10: Aircraft response from a post stall mode using Improve High Order Sliding Mode Controller

Bounds Fictitious controllers	V1(Thrust)	V2(dele)	V3(dela)	V4(delr)
Min	-12336.4	-910.325	-131551	-1.66*10$^($6)
Max	17004.7	615.636	131551	1.66*10$^($6)

Table 5.1: bounds on the magnitude of the fictitious controller

Bounds Gains	K1	K11	K2	K22	K3	K33	K333	K4	K44
Min	0	0	0	0	0	0	0	0	0
Max	548.901	403.271	22670	19494	7.30409	2.17	13.4052	4068.3	4080.4

Table 5.2: bounds on the gains for the linear fictitious controller

In order to compute all the maximum gains associated to each fictitious controller for the high order controller that allow us to stabilize the integrators obtained from input/output decoupling, we maximized the sum of those values because we know that the are position so maximize the sum is equivalent to maximized each one of the values above.

The next step after the bounds of the controllers is obtained is to simulate the system within the bounds to find the optimal values of the bounds that would give the require performance. Because we are using an LPV model, we can easily slightly vary the parameters with the set up and compute the others on the controllers. Apply the same the technique for other control techniques such as adaptive for tuning while knowing the bounds on the required controllers gains.

Improve results on the responses for the High Order Sliding Mode Controller and Feedback Linearization where we actually allow saturation. The results below shows progress on the thrust reduction but a set of simulation for improvement of the results still under investigation. Also there also an impressive minimization of the thrust due to the fact that we took into consideration the moment arm between the thrust axis and the actual x-axis of the aircraft.

After Inequalities construction and computation of the bounds on the controllers, we have improvement on the controls even though we still working on making sure saturation does not cause a problem in the recovery process.

5.5 Analysis for validation of Recovery Strategies

In this particular section of the chapter, we are making an analysis of the different techniques of recovery based on the load factor. The structural integrity of the aircraft should be maintained

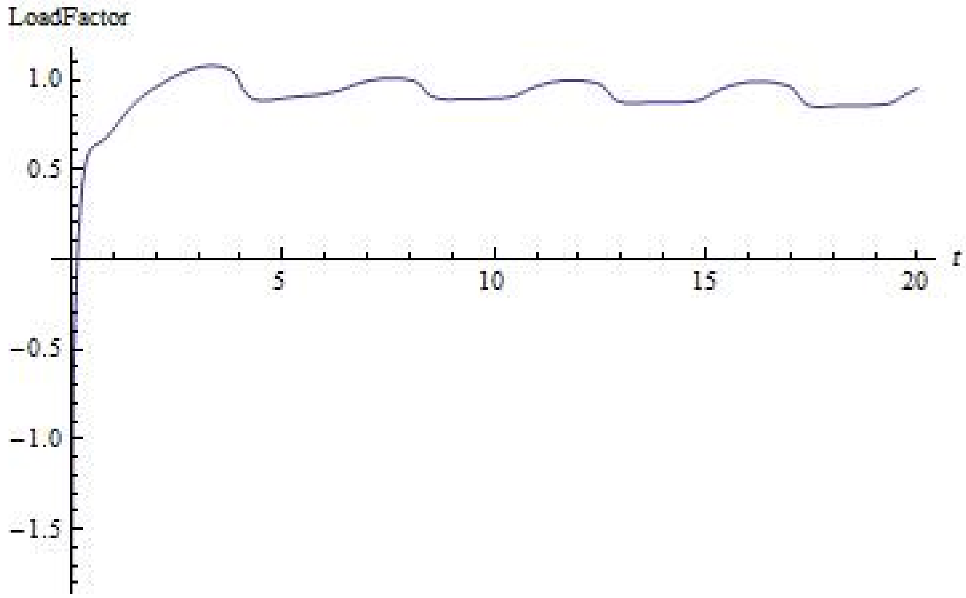


Figure 5.11: Evolution of Load Factor during the recovery process with High Order Sliding Mode Control

during the recovery process and as a result it should not exceed a certain amount of load factor. we plotted and analyzed the time evolution of the load factor throughout the evolution of the recovery for each technique. Certainly a measure of a technique should how close the process time evolution is to the normal g-maneuvers process. By definition we have ratio of the lift to the weight as measure of the structure evolve over time when the aircraft is on its flight mission or flight maneuver. We know that the load factor should not be less than 2.5 and should not exceed 3.8 for the airplane to maneuver safely and return to the normal flight regime and also should vary linearly with speed between the cruise speed and the dive speed. By definition, the load factor is the ratio of lift to the weight

$$n_{load} = \frac{Lift}{Weight} \tag{5.5.1}$$

Based on the paper from Austin et al [92], you can see that the AirSTAR aircraft has an impressive load factor protection where the maximum values for the load factor vary between 4g and 6g which show satisfaction of the recovery control laws designed and cited in this thesis. With the procedure describes in the Appendix, we can easily compute the load factor based on the lift that can be extracted from the given aerodynamic data. Taking the steps, I extracted the steady states of all the states with the maximum thrust and others controls and I obtained a value between 4g and 6g which shows a satisfaction in the recovery process. My only concern was around the first few minutes during the recovery and a better simulation and adjustment of the parameters can help fix the problem. The recovery control laws designed sofar should be implemented in real time to confirm their validity.

Observation of the boundary Departure from Controlled Flight

The end of this section highlight the Weissmann curve which is the comparison between the Lateral Control Departure Parameter (LCDP) criteria and $C_{n_\beta Dyn}$ which a measure of avoiding the aircraft to depart to spin because of its sensitivity during the recovery process. we know that the Weissmann condition would divide the region is small pieces where certain region should be avoid, then we would conclude that the aircraft may no depart to spin during that process of restoring the aircraft into the safe mode already compute in one of the previous chapter. In this section we also added the algebraic condition that determine the aircraft in falling and leaf condition which would a significant role in analyzing and designing controllers for recovery especially for military aircraft [96, 97].

$$\begin{aligned} LCDP &= C_{N_{beta}} - C_{L_{beta}}\left(\frac{C_{N_{dela}}}{C_{L_{dela}}}\right) \\ C_{nbetadyn} &= C_{N_{beta}} * Cos[alpha] - \left(\frac{I_z}{I_x}\right) * C_{L_{beta}} Sin[beta] \end{aligned} \tag{5.5.2}$$

Aircraft in Falling and Leaf algebraic condition A set of key algebraic parameters are used here to emphasize the prediction of the falling leaf bifurcation problem sofar. Here we do not confirm the validity of those boundary parameters but used them in this thesis for future investigation on the falling bifurcation. Before that we see the falling leaf case as the periodically stable flight condition much like a spin. Falling leaf occurs when the rolling and yawing moments are in phase and act such that roll encourages yaw and vice versa [52]. Two algebraic conditions allow me to isolate region of the state space where the aircraft can depart from controlled flight to falling and leaf mode.

SRYP: Synchronous Roll-Yaw Parameter:

$$SRYP = \frac{\frac{C_{n\beta}}{C_{l\beta}} + \frac{I_{xz}}{I_{xx}}}{\frac{I_{zz}}{I_{xx}} + \frac{I_{xz}}{I_{xx}}\frac{C_{n\beta}}{C_{l\beta}}} \tag{5.5.3}$$

Dutch-Roll Stability Parameter:

$$C_{n\beta_{Dyn}} = C_{n\beta}\cos(\alpha) - \frac{I_{zz}}{I_{xx}} C_{l\beta}\sin(\alpha) \tag{5.5.4}$$

These are algebraic condition that should be solved to determine exactly the condition of depart from controlled flight to falling and leaf mode and it would be clear that if the following inequalities are true then the aircraft depart from control flight to falling and leave. The GTM case would be studied is this thesis.

$$C_{n\beta_{Dyn}} > 0 \ and \ SRYP > 0 \tag{5.5.5}$$

The curve below would indicate the case of the aircraft generic transportation model.

5.6 Conclusion

In conclusion, this chapter explores the recovery strategies for an aircraft in a post stall regime.Before exploring all the strategies, we survey the different behavior in a post stall regime and generalized the procedure of recovery using dynamic feedback because if you can use a static output feedback as a control law and extend the model by putting integrator in from of input channels then the dynamic feedback is obtained. we also used optimal strategy to design an optimal control for the aircraft recovery purposes. Then we expand the post stall recovering techniques by looking at the feedback linearization and High Order Sliding Mode controllers. At the end, we analyzed each strategy by observing the evolution of the load factor and we conclude that the optimal controller is better because it does not add too much stress on the vehicle and loss of altitude in minimized. The high order sliding mode case works well and performs a rapid action although a brutal at the beginning of the recovery but recover to a 1g as expected with better minimization of the loss of altitude. we ended the chapter by looking at the Weissmann strategy of avoiding controlled departure to the spin while recovering and we think based on the analysis performed by Weissmann, we should not expected a departure to spin while performing recovery. Also the Weissmann [99] analysis plays a capital role in the next chapter where decision and transitions during flight operation are necessary.

6. Aircraft Hybrid Fault Tolerant Control Systems: Multiple Models Approach

In general the prevention and recovery strategy would be managed by a high level controller known as a supervisor. Its major goal would be to learn as much as possible from the aircraft behavior before choosing the right controller to manage the remaining system without overwrite the pilot's decision. In this chapter,we formulated the prevention and recovery strategy as a Hybrid Fault Tolerant Control System(HFTCS) where several scenario are taken into consideration and a Model Predictive Control(MPC)technique is used to learn from the behavior of the system each sample times before restore it. The benefit of using these technique is gained from the advanced microprocessor technology and also the used of embedded controllers for rapid action. Rethinking the conception of modern autopilot as a multiple models approach should not come as a surprise. First because the concept is been approach by several adaptive researchers over a decade [13, 12, 100] but the downside of this approach wasn't as successful as expected for a number of fundamental reasons. My contributions in this chapter comes at two majors points:Implementation of the hybrid strategy algorithm and also the algorithm that check a set of conditions within sampling period.

1. In aircraft flight control, switching between controllers based on different models can result in instability of the overall system [101].

2. Multiple model adaptive control approaches has proved to be sensitive to uncertainties associated with parameters.

3. During a catastrophic failure, a rapid action is often needed and due to online parameter adaptation, delay may cause a plane crash.

4. Previous approaches rely linear controllers which has proved inadequate because there are flight regimes in which linear controllers may not be appropriate - especially maneuvering near high-angle-of-attack. The control surfaces may saturate with not enough room to pull the nose down and the aircraft will stall [23].

5. Environmental conditions have also been a critical factor because assumptions were always made for almost how far can the aircraft resists which was not always valid to overcome major disturbances such as the strong cross winds, wake turbulence, icing etc...

Accounting for these points in advanced flight control systems is challenging because the control system also needs to address the basic goals of performance and flying qualities. The section below considers these points and attempts to investigate whether answers do exist. Two great points would make an advanced smart autopilot from the integration of the key highlight above.

1. Allow pilots to have more display for decision making in instantaneous events
2. Design an internal supervisory management for the overall vehicle without overriding the pilot's decision.

6.1 Issues with Aircraft Hybrid Fault Tolerant System Implementation

In analyzing the issues cited above, we would provide answers to problem and how they should be incorporated or cascade into the final design before final real time test. There is an overall fact to be admits that is the tremendous advanced in microprocessor which has spurred the entire aerospace community converting every research topic into embedded processors. In addressing those above points, we admit that the online processing speed may no longer be an issue.

Before we focus on other issues, we would like to highlight the stability issues which appear to be at the heart of multiple models adaptive control, Models with multiple controllers switching and hybrid systems. A number of researchers during the last decade have focused their research on improving the stability while switching between models or while switching between controllers. Because our intent is to use multiple models to address the issues above, we would rely on two keys point both related to the Lyapunov Concept.

1. **Multiple Lyapunov Functions [102]**

   Before we also look at the stability issue, we are forced to define the notion of hybrid system in this context and the next section would focus on explicitly describes models and the approach to partition the state space.

   Definition 6.1.1 (Hybrid Systems). A continuous time controlled hybrid system is defined as:

$$\begin{aligned}
&\dot{x} = f(x, q, u) \\
&q^{+} = \sigma(x, q^{-}) \\
&y = g(x, q, u) \quad u \in \mathbb{R}^m; y \in \mathbb{R}^p; z \in \mathbb{R}^q \\
&z = h(x, q) \quad x \in \mathbb{R}^n; q \in Q = \{1, ..., n\}
\end{aligned} \tag{6.1.1}$$

Where Q is the set of discrete mode or dynamical behavior associated to the complete systems,$\sigma(.,.) : \mathbb{R}^n \times Q \to Q$ defines the servo signal generator used to select the subsystem under operation. We also have the set of outputs and the set of errors.

Theorem 6.1.1 (**Multiple Lyapunov Functions**). *Suppose we have the following candidate Lyapunov Function: V_i, $i \in \{1, ..., n\}$ and a set of closed loop vector fields: $\dot{x} = f_i(x)$ with $f_i\,(0) = 0$ Let the set of all switching sequence associated with the system.If for each $s \in S$ we have that for all i, V_i is Lyapunov like for f_i and $x_S(.)$ over $S|_i$ then the system is stable in the sense of Lyapunov*

2. **Stabilization by Switching Among Controllers [98]**

   The section almost repeats part of the precedent chapter and I want to emphasize the section here because it allows me to crack the real problem associated to the aircraft loss of control. In reality, across section around the trim the zero dynamic is structurally unstable the Gain scheduling if not properly interpolated, the aircraft would lost its control effectiveness and as we all know the aircraft is out of control. The Gain scheduling community did not paid attention to the partition of the state space before designing controllers which one of the main reasons for aircraft lost of control. In this section is particularly important in a sense it emphasize the partition of the state space and at the same time, at the same time gives the sufficient condition for stability of the overall system once cascade. We also define the switching function which would allow be to switch across surface of the state space.

   Theorem 6.1.2 (**Switching Among Basic controllers**). *Given the following dynamical aircraft nonlinear dynamical system such as it is the case with Linear Parametric Varying Models.*

$$\begin{aligned} &\dot{x} = f(x, u) \\ &y = h(x) \quad x \in \Re^n; u \in \Re^m \end{aligned} \tag{6.1.2}$$

And let's define the basic controllers:

$$\begin{aligned} &u_i(x) = k_i(x),\ k_i(0) = 0,\ \forall i = \{1, ..., N\} \\ &and \quad f(0, k_i(0) = 0,\ \forall i. \end{aligned} \tag{6.1.3}$$

Suppose that the vector functions $f(x, k_i(x))$ are at least once differential and that

$$\begin{aligned} &\dot{x} = \tfrac{\partial f(x, k_i(x))}{\partial x}|_{x=0} + g_i(x) = F_i x + g_i(x) \\ &\lim_{|x| \to 0} \tfrac{|g_i(x)|}{|x|} = 0 \quad \forall i \in \{1, 2, ..., N\} \end{aligned} \tag{6.1.4}$$

Then the system above is locally stabilizable by switching between basic controllers if there exists a matrix

$$\begin{aligned} &P = P^T > 0 \\ &\tau_i \geq 0;\ \sum_{i=1}^{N} \tau_i > 0\ \ s.\ t \\ &\sum_i \tau_i (F_i^T P + P F_i) = -Q \quad Q = Q^T \end{aligned} \tag{6.1.5}$$

The proof of the theorem can be found in [100].

The above theorem turns out to be very interesting because of the realτ_i value can be used as binary variables and we will always be sure that at least one case holds which looks promising for stabilization of the overall system as hybrid systems. The drawbacks of others approaches to hybrid systems or adaptive controllers are that they do not pay attention to the behavior of the zero dynamic or the behavior of the sliding dynamic for those who are using variable structure control. At the end for implementation, we would make sure that the zero dynamic are well behaves or allow a switching strategy for the stability of the zero dynamic. The procedure would be outline throughout the implementation.

Once stability between switching controllers is obtained, the next problem is what if they are errors in the evolution of the system or if the pilot makes error during flight by not paying attention to the stall prevention warnings? Then the aircraft fall into a post stall regime and in this situation, we designed an offline controller that could be used for recovery purposes. The recovery controller would be activated with alternative power because it sound like there is a need for external power for better recovery from an instable regime to the safe set that was precomputed in the third chapter. An illustration can be view in the safe set computation because they describe the flow of safe trajectories.

More than just recovering from an unstable regime or allow maneuverability near critical regime, from the computation of the safe set, we also derive the controller that would allow us to stay within the safe set and also in the second chapter evaluate the impact of stuck elevator to the geometry of the safe set. Once its nature is found, then a reconfigurable controller is then compute with the satisfaction that it would get back to the domain even at reduce performance.

The last point of this section regard the environmental condition such as icing condition which is not fully address in the context of this work but would be examined and cascade it into the complete system to avoid undesirable skidding on the runway and or stuck elevator due to ice.

At the end, we can see how some of the issues regarding advanced digital flight-by-wire can be addressed using multiple models and mange under the context of hybrid system where a supervisory controller is used to choose the right controller for the proper flight maneuver. I'm now assessing the description of each mode with the satisfaction that enabling states can be reached easily with the finite reach ability problem already solved [37]

6.2 Advanced Aircraft Autopilot as Hybrid Fault Tolerant Control using Hybrid System with multiple Models

This particular section of the thesis underscores the variables and the key hybrid aircraft dynamical modes. Before describing the dynamical mode, the motivation behind using the hybrid system instead of just parameter adaptive control comes from zeroing my outputs and oberserving the singular sheet that partition [63] the state space as shows the graph below. We know that across contingent sheets, the same controller may not be appropriated and in the neighborhood of adjacent sheets, the aircraft is susceptible of loosing control.

In viewing the curve above, we understand the importance of stabilizing the zero dynamics while going from one trim condition to the other trim condition in the state space. Because going from one trim condition to the other trim condition may cause instability of the complete system which is why partitioning the state space becomes a very important process.

Once we motivated ourselves enough on the reason for using hybrid system to model the advanced autopilot control system, it's time for us to concentrate on describing the different aircraft dynamical models that would participate in the first implementation of the complete systems.

First step in the description is to understand the equilibrium manifold which is the set of outputs to be regulated by the pilot during flight maneuver. For a smooth equilibrium manifold, we can have a nominal model made of gain scheduling. When we look at the nominal mode, we would take into account the longitudinal and lateral controlled flight and also the coordinated turn. That would be the main nominal mode of the aircraft systems.

Near critical points or in a post bifurcation (post stall) the aircraft would operate with a different controllers and the controller is designed offline and embedded into the system for fast actions in the recovery process.

The third dynamical mode here falls into the category of stuck actuators or deflection surfaces where smooth reconfigurable controller may not have enough control effort to steer back the aircraft into the nominal flight regime. Such position are actually evaluating offline and embedded a reconfigurable nonlinear controller such that the aircraft goes back into the normal safe set where an existing trajectory is actually safe.

The fourth dynamical mode falls into the category of flight where environmental conditions are taken into account such as icing condition. Sofar for the first investigation of the validity of the concept we emphasize the first dynamical modes with possibility of extension for better operation. Below is a description of the mathematical model as it should be conceived before implementation.

underlineMathematical Description of the Hybrid Fault Tolerant Control System Model.

The aircraft hybrid fault tolerant control systems associated to the*loss-of-control* prevention and recovery would be presented as an automaton:

$$A = (X, Q,\ V,\ f,\ Inv,\ E,\ G)$$

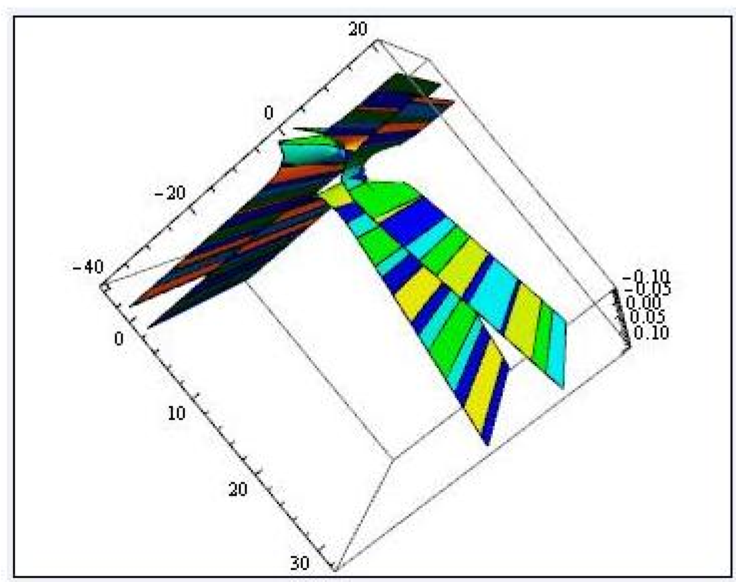


Figure 6.1: Singular sheet that partition the state space

Where we have the following definitions:

1. $X \subseteq \Re^n$ is the continuous state space of the system Q is the finite set of modes that defines the discrete state space of the system. In my approach, for a start I have three different models where the controllers are designed offline for critical conditions such as the post stall situation, actuators failures, stuck deflection surfaces and others.

$$X = \{\varphi, \theta, \psi, x, y, z, p, q, r, V, \alpha, \beta\}$$

$$Q = \{q_1, q_2, q_3\}$$

2. $V = U_c \times U_d$ Defines the input space of the system, respectively continuous and discrete (Our discrete input here would be binary variable)

3. The continuous control inputs are the thrust and the deflection surfaces while the discrete input is the servo signal that generate the transition

4. $f : Q \times X \times U_c \times U_d \rightarrow X$ assigns to each discrete location a continuous vector field $f_q(x, u_c, u_d)$ Where for $q = 1$ we have the nominal and for $q \neq 1$ we have the impaired systems.

5. $Inv : Q \rightarrow 2^X$ assigns to each control location a region of attraction recognize as an invariant set. Or a specific region of the state space that can be computed using the following formulation:

$$I^*(x) = \underset{i \in \{1,.......,N\}}{\arg\min} \; x^T[(A + BK_i)^T P + P(A + BK_i)]x \tag{6.2.1}$$

$I^*(x)$ is the switching function defined by the above equation and is a multi value function defined on a set $S_\omega \subset \Re^n$ which is a non dense subset of the state space. That non dense set is defined as the union of the following set.

$$S_w = \bigcup_{j,k \in \{1,..,N\}} \{x \in \Re^n | 2x^T PB(K_j - K_k)x = 0, \; j \neq k\} \tag{6.2.2}$$

6. $E \subseteq Q \times U_d \times Q$ Is the set of discrete transitions between modes The benefit of this is that all the transition can be evaluated offline because of the finite set of discrete modes. This can done easily using logical operations

7. $G : E \to 2^X$ Assigns to each $(q,\, u_d,\, q') \in E$ a guard set $G(q,\, u_d,\, q')$ such that $G(q, u_d, q') \cap Inv(q) \neq \emptyset$ The guards set are evaluated by understanding the behavior of the aircraft and using some algebraic conditions cite above such as LCDP, $C_{n\beta dyn}$ to enable transition between discrete modes.

The guard sets are use to enables transitions as long as the next dynamical mode is reachable and the appropriate discrete input is supplied. In the aircraft context where all states are not always available, the continuous state estimation and discrete state estimation becomes an important factor to take into consideration during the construction of the hybrid observers. The computation of the safe sets and guard sets will be critical in the synthesis of controllers that guarantee constraints evolution within the safe sets. Finite time reachability becomes also an improvement and a consistent pattern for enabling switching in the transition. At the end we expect the mathematical model to look:

$$\begin{aligned} &\dot{x} = f(x, q, u) = \sum_{i=1}^{s} \delta_i f_i(x, u_i(x, \delta_i)); \quad \delta_i \in \{0, 1\} \\ &y_i = h(x, \delta_{i-1}(x, u_{i-1}(x, \delta_{i-1}))) \end{aligned} \tag{6.2.3}$$

In this formulation, we used the discrete state equation and the performance equation to compute the trim states and the controls and use those values to determine the next value of the measure sensor or the position of the flight vehicle in space. Based on that information, we can determine which dynamical may take over. Within each sampling time, it is important to perform a set of operation before making a decision whether or not a particular dynamical mode might be suitable for the next move. Such test may be based on the controllability and observability of the actual approximated model, the health of a particular component in the system and also availability of a particular control effort to manage the next mission. All these assessment can be made within the time period because the processors speed is manageable. A sample algorithm is describe in the appendix and it was used in Dongmo et al [46] to manage the controllability of the system as we approaching the stall point in space. The entire system will be managed by a high level controller called supervisory controller[119,120,121]

6.3 Algorithm for Implementation of the AHFTCS

The algorithm outline here suppose that we have to generate trajectory and follow that trajectory in a piecewise control fashion which means on the sample time basis, a number of test would have to be made such as the controllability, observability, detectability and stability of the zero dynamics as well. A chosen controller would be based on a series of this test and the following is the required algorithm that should be implemented on a real time. Given the above aircraft dynamical system, the following algorithm would be used to compute the trim equilibrium point, Find the linearize model, check its feasibility, design the controller and used that controller to figure out the next point and at the same time we would also test the stability of the zero dynamic as well, check the stability of the zero dynamic appears to novelty in the following algorithm with the assumption that the aircraft dynamic is converted into the Brunowski form where the zero dynamic can be easily extract test at the same time as the dynamic equation before the next sample.

$$\begin{aligned} &x(k+1) = f(x(k), q, u(k)) = \sum_{i=1}^{s} \delta_i f_i(x(k), u_i(x(k), \delta_i)); \quad \delta_i \in \{0, 1\} \\ &y(k) = h(x(k), \delta_{i-1}(x(k), u_{i-1}(x(k), \delta_{i-1}))) \end{aligned} \tag{6.3.1}$$

Algorithm

Given x_k, y_k, u_k *and* n_1, n_2 *and* $f, f_x, f_y; h_x, h, h_y$

Determine

$$x_{k+1}; y_{k+1}$$

Using the following procedure

1. Compute y_k^+ from the equilibrium equation for the trim condition $h(x_k, y_k^+, u_k) = 0$ using n_1 iteraction of Newton's method, starting with y_k

2. Compute x_{k+1}, y_{k+1} from

$$\begin{aligned} x_{k+1} - x_k - \tfrac{1}{2} h(f(x_k, y_k^+, u_k) + f(x_{k+1}, y_{k+1}, u_{k+1})) &= 0 \\ h(x_{k+1}, y_{k+1}, u_{k+1}) &= 0 \end{aligned} \tag{6.3.2}$$

   Using n_2 iteraction of Newton's method starting with x_k, y_k^+

(a) Define $z = (x, y)$

$$
\begin{aligned}
F(z) &= (x - x_k - \tfrac{1}{2}h(f(x_k, y_k^+, u_k) + f(x, y, u_k)), g(x, y, u_k)) \\
F_z &= \begin{bmatrix} I - \frac{h}{2} f_x(x, y, u_k) & \frac{-h}{2} f_y(x, y, u_k) \\ h_x(x, y, u_k) & h_y(x, y, u_k) \end{bmatrix}
\end{aligned} \tag{6.3.3}
$$

(b) $Newton(F(x, y, u_k), F_z(x, y, u_k), (x, y), (x_k, y_k), n_2)$

The algorithm is implemented using the computer tools cite below. Integration of that computer tools will then be used to determine the next trim condition in flight before tries to steer the aircraft near that trim point. The algorithm is flexible because within each time period, we can check a few conditions before we can move to the next step.Those conditions are based on the status of the next trim point whether or not it can be associated to an irregular situation in flight

The following implementation is used for identification and to compute the next position at which we want to be by using the previous control input to the system.

6.4 Computer tools use in the design and analysis of the thesis

1. ProPac/Mathematica are used for symbolic computation
2. Simulink/Matlab is used for the implementation of the dynamic and offline design controllers
3. Stateflow/Matlab is used for implementation of the logic or the supervisory controller that will manage the complete system.
4. The overall autopilot system would be implemented in Matlab environment for real time test as future research

6.5 Conclusion

In conclusion this chapter was outline for future research and does not participate explicitly in the thesis. It just plays a role for guidance of future research in the idea where the loss-of-control appears to be a major factor or where a high level of automation should be implemented. In fact, It just shows how the overall system would appears to be at the end once every major piece of the critical part of the equilibrium manifold is studied and cascade into the overall system. The

Autopilot Hybrid Fault tolerant control system would be extended in adding environmental condition in which icing plays a significant role. In the end the design autopilot would be safe and capable of autonomously make decision without overriding the decision of the pilot in instant event.

7. Conclusion and Future Research

7.1 Conclusion

In conclusion the thesis presents, the novelty in the design of advanced autopilot where a few critical points are extracting and controlled offline with type of control laws. In our case, we approached the problem of improving the autopilot design by first looking at the prevention case. Prevention in this case means compute the maximum safe set within the original flight envelope using Hamilton-Jacobi Equation formulation.

The third chapter presents the overall idea of how to compute the safe set in detail and shows an example of a reduced longitudinal aircraft dynamic, the phugoid mode where we assume the flight path angle is very small. we also extended the concept by looking at the case in which there is a stuck elevator and address the shape of the safe in the same chapter. In that chapter also, we ceased the opportunity to design a switching controller with the assumption that the aircraft would be restored within the safe set where there is a possibility of a trajectory flowing towards a known and existing trim points. The procedure is illustrated using a two points boundary value problem.

Chapter four plays a central role in the thesis where regarding previous approaches, where controllers were designed for recovery using thrust vectoring, dynamic inversion methods, siding mode, it is well known that the zero dynamic is structurally unstable and classical methods may not worked properly. The particularity of this work falls into two majors categories.

First we approached the design of a recovery controller in a post bifurcation where the loss -of-control is due to the sensitivity of the zero dynamic because of the fold nature of the equilibrium manifold using optimal control where we extended the model and formulated the recovery problem using Hamilton-Jacobi Bellman equation. The problem was approach in two steps first design a regulator controller and then attempt to minimize altitude lost while regulating target set point within the safe set.

This approach uses feedback linearization as a tool to derive the high order sliding mode controllers for aircraft in the post stall recovery process. While using High Order Sliding Mode Controller, we tested the feasibility of the controller obtained using feedback linearization, it works but with the thrust out of the normal bounds. The problem of control bounds was approached by other researchers in the field by adding extra source of power during the recovery. In our case, we simply reformulate the problem using the known bounds on the actual controllers to compute the bounds on the fictitious controller and latter on bounds on the gains for the high order controller that stabilize the set of integrators. In order to validate the feasibility of our controllers, we observed the time evolution of the load factor associated to the aircraft in order to make sure that we have the vehicle performing complete action while remaining assemble. Sofar, we have impressive response which shows that the recovery process can be tested in real time without questions.

7.2 Future Research

As future research, at the time where I'm completing my thesis, the entire project has an impressive goal which is overall improvement of autopilot for safety in several aspects. Search for ways to address the envelope protection of the aircraft in flight which is covered in this thesis, also solve the problem of high gains by computing the bounds which is also covered in this thesis despite the fact that the bounds are overestimated. The most important problem which is addressed in the loss-of-control problem is the attempt to design automatic control laws for recovery. The problem is also addressed in this thesis.

Before the final integration, we started during the introduction of this thesis deriving equilibrium manifold using Quantifier Elimination technique which to my knowledge still have some computation problem because of the curse of dimensionality. In a future, the technique can be explored for better analysis of flight control system in a multi-objective context and then also used to address the bounds on the controllers gains using specific parameters for robustness analysis.

The final problem which is not addressed in this thesis is the implementation of the overall system in a complete system that we labeled Hybrid Flight Tolerant Control System where a few points are already tackled and the remaining points are outlined in chapter six. Future research in the development of this topic would be to integrate the controller already designed and test them for real time application. Before real time application, they have to be simulated in a real time software environment such as Matlab before real world integration. Moreover the idea of multi-models hybrid system has to be fully investigated and implemented as a solution of the LOC control.

Appendix

GTM Aircraft Parameters

It's true that there exists several models of the GTM aircraft used nowadays but the model that we used had the following as parameters:

$$S = 5.9018 ft^2$$
$$x_{ref} = 0.15$$
$$b = 6.8488 ft$$
$$\bar{c} = 0.9153 ft$$
$$I_{xx} = 1.327$$
$$I_{yy} = 4.254$$
$$I_{zz} = 5.454$$
$$I_{xz} = 0.12$$
$$m = 1.5416$$
$$\rho = 2.37E^(-3)(slug/ft^3)$$

GTM Dynamical Equations

The aircraft dynamic that I used follows the pattern of general equation of motion of a robotic system, model using Euler-Lagrange formulation.

$$\dot{q} = V(q)p$$
$$M(q,p)\ddot{p} + C(p,q)\dot{p} + F(p,q,Th,dele,dela,delr,spoilers,flaps) = Q$$

Where $q = (phi, theta, psi, x, y, z)$ represents the Euler angular angles for the orientation of the aircraft in space and the position of the aircraft with respect to the earth reference frame. $p = (p, q, r, u, v, w)$ represents the Euler angular rates and the translational velocities along the axis. $F_p = \{L, M + lt * Th, N, F_x + Th, F_y, F_z\}$ Where $\{L, M, N\}$ represents the dimensionless aerodynamics moment equation in this context, obtained as polynomial fitting with aerodynamic data and the wind coordinates. $\{F_x, F_y, F_z\}$ represents the aerodynamic forces along the axes and are also obtained using polynomial fitting and the wind coordinates and the deflection surfaces. l_t is the perpendicular distance between the thrust axes and the x-axis of the aircraft in flight.

$$Q = \{Wxcg\cos(phi)\cos(theta), Wxcg\cos(theta)\sin(phi), W\sin(theta),$$
$$W\cos(theta)\sin(phi), W\cos(phi)\cos(theta)\}$$

Q represents the external forces converted into the wind references frame. The velocities are mapped using the following transformation:

$$V(q) = \begin{bmatrix} 1 & \sin(phi)Tan(theta) & \cos(phi)\tan(theta) & 0 & 0 & 0 \\ 0 & \cos(phi) & -\sin(phi) & 0 & 0 & 0 \\ 0 & \sec(theta)\sin(phi) & \cos(phi)\sec(theta) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(phi)\cos(theta) & I1 & J1 \\ 0 & 0 & 0 & \cos(theta)\sin(psi) & I2 & J2 \\ 0 & 0 & 0 & -\sin(theta) & \cos(theta)\sin(phi) & \cos(phi)\cos(thet \end{bmatrix}$$

$$I1 = -\cos(phi)\sin(psi) + \cos(psi)\sin(phi)\sin(theta)$$
$$I2 = \cos(phi)\cos(psi) + \sin(phi)\sin(psi)\sin(theta)$$
$$J1 = \sin(phi)\sin(psi) + \cos(phi)\cos(psi)\sin(theta)$$
$$J2 = -\cos(psi)\sin(phi) + \cos(phi)\sin(psi)\sin(theta)$$

$M(p,q)$ *and* $C(p,q)$ Represents the inertia matrix and the coriolis and others friction components acting internally in the aircraft. The outputs variables that the pilot usually tries to regulate in flight are:

1. The speed $V = \sqrt{u^2 + v^2 + w^2}$
2. The flight path angle: $theta - alpha$
3. The heading: $\xi = phi + beta$
4. The bank angle

$$\mu = \arccos\left(\frac{\cos(alpha)\cos(theta)\cos(phi) + \sin(alpha)\sin(theta)}{\cos(theta - alpha)}\right)$$

Depending flight mode, certain outputs can be more important than the others, In a coordinates turn there is an algebraic condition that should always be taken into consideration when deriving the control law: $Lift * Cos(\mu) - W = 0$

The following transformation is very important in converting the coordinates from the body references to the wing reference frame where most analysis and design are conducted.

$$\begin{aligned}
u &= V\cos(alpha)\sin(beta) \quad & V &= \sqrt{u^2+v^2+w^2} \\
v &= V\sin(beta) & beta &= \arcsin(v/V) \\
w &= V\sin(alpha)\cos(beta); & alpha &= arc\tan(w/u)
\end{aligned}$$

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = H_W^B \begin{bmatrix} -D \\ SF \\ -L \end{bmatrix}$$

GTM aircraft LPV Generation

Deriving advanced control systems would require the understanding the equilibrium manifold because they are the set of the regulated outputs. In designing flight control system, I focus on deriving two different models for my thesis. One model is just an extension of the other nominal linear model usually obtained from Taylor expansion around a given equilibrium point. Then I truncated to the highest order terms which in particular for my case is 2 and 3 for the extended model.

$$\begin{aligned}
f(x_0+\delta x, u_0+\delta u) = f(x_0,u_0) + \tfrac{\partial f(x_0,u_0)}{\partial x}\delta x + \tfrac{1}{2}\tfrac{\partial^2 f(x_0,u_0)}{\partial^2 x}(\delta x)^2 + \\
+\tfrac{1}{3!}\tfrac{\partial^3 f(x_0,u_0)}{\partial^3 x}(\delta x)^3 + \tfrac{\partial f(x_0,u_0)}{\partial x}\delta u + \tfrac{1}{2}\tfrac{\partial^2 f(x_0,u_0)}{\partial^2 x}(\delta u)^2 + hot
\end{aligned}$$

$$h(x_0+\delta x, u_0+\delta u) = h(x_0,u_0) + \frac{\partial h(x_0,u_0)}{\partial x}\delta x + \frac{\partial h(x_0,u_0)}{\partial u}\delta u + hot$$

That particular formulation plays a significant role in studying the zeros at the bifurcation points where we can potentially decouple the simple models with one stable and the other unstable. This fact of unstructured zero dynamic is favorably the most interesting reason of aircraft *loss-of-control.*

The second model is also important and plays a significant role in flight control system especially in hybrid control strategy where we have to used controller with different structure depending on the flight regime and the require maneuvers. It also plays a significant role in addressing the controllability and observability near critical domain. In the era where there are significant progresses in the processing speed of processors, then piecewise feedback becomes increasingly useful both in control system design and digital implementation of the flight -by-wire with the emphasis on addressing every single aspect of the flight domain. Because I'm focusing on the stall and the post stall regime where it's particularly useful to use the Linear Parametric Varying model to address the issues and make decision on the type of controllers. The principle of derivation of LPV model is outlined and is structurally important in addressing control system design by analyzing its input/output structure and is also useful for the purpose of designing high level supervisory control.

1. Construct the equilibrium set:

$$ES = \{(x,u,\mu) \in \Re^{n+m+k} | f(x,u,\mu) = 0 \, and \, h(x,u,\mu) = 0\}$$

2. Defines a set of k-coordinates around at each point point $(x_0, u_0, \mu_0) \in ES$

3. if

$$\bar{x} = (x,u,\mu) \; and \; \bar{F}(\bar{x}) = \begin{cases} f(x,u,\mu) = 0 \\ h(x,u,\mu) = 0 \end{cases} \quad the \;\; goal \;\; is \; to \; construct$$

$$(x(s), u(s), \mu(s)) : \Re^k \to \Re^{m+n+k}$$

The following relationship must be satisfied:

$$D_{\bar{x}}\bar{F}(\bar{x})\frac{\partial \bar{x}}{\partial s}ds = 0 \Rightarrow \frac{\partial \bar{x}}{\partial s} \in \ker(D_{\bar{x}}\bar{F}(\bar{x}))$$

From the equation above, we can always compute a basis of the kernel such that:

$$\frac{\partial \bar{x}(s)}{\partial s_i} \in span\{\gamma_1(\bar{x}), ..., \gamma_k(\bar{x})\}, \quad i = 1, ..., k$$

$$and \, the \;\; solution \;\; of \;\; the \;\; above \;\; equation$$

$$\frac{\partial \bar{x}}{\partial s_i} = \gamma_i(\bar{x}); \quad i = 1, ..., k$$

$$the \;\; flow \;\; defined \;\; by \;\; the \;\; differential \;\; equation \;\; is \;\; \varphi_i^{s_i}(\bar{x})$$

$$and \;\; satisfied$$

$$\frac{\partial \varphi_i^{s_i}}{\partial s_i} = \gamma_i(\varphi_i^{s_i}) \quad with \;\; \varphi_i^0(\bar{x}) = \bar{x}$$

For a k-dimensional regular manifold that is parametrically characterized by the mapping

$$\bar{x} : \Re^k \rightarrow \Re^{N+k}$$
$$\bar{x}(s) = \varphi_1^{s_1} \circ ...... \circ \varphi_k^{s_k}(\bar{x}_0)$$

From the above formulation, we can then derive an equilibrium surface and then derive an LPV model based on the basis of the kernel of the jacobian defined above.

Algorithm used for the recovery Process in the NonLinear Smooth Controller case.

Implementation of the algorithm with Simulink

Hybrid Implementation

Algorithm For Stabilization and Regulation

```
ExtNLController[NLDync_List, BifurPt_List, QQ_List, RR_List, Xvect_List, Contrl_List
 Module[{Stvect, ExtNL, AA, BB, K1, P, Eigs, V0, LinCtrl, Ctrl1, Ctrl2, VectorList, Vec
    VectCoef1, VectCoef2, c1, c2, c3, c4, c11, c22, c33, d11, d12, d13, d22, d23, d24,
    d33, d32, d34, d44, d42, d43, h1, h2, h3, h4, h11, h22, h21, h23, h33, h31, g11, g12,
    g13, g22, g23, g24, g33, g32, g34, g44, g42, g43, y1, y2, y3, y4, sys, Sys2, ff, f2,
    f3, Coeff, CFF, CCf2, Equans2, Sol2, Contrl1, Contrl2, FstOrdCont, SecOrdCont},
  ExtNL = Truncate[(NLDync /. Inner[Rule, Join[Xvect, Contrl],
       (Join[Xvect, Contrl] - BifurPt), List]), Join[Xvect, Contrl], n];
  ExtNL = ExtNL - ((ExtNL) /. Inner[Rule, Join[Xvect, Contrl],
       ConstantArray[0, Length[Join[Xvect, Contrl]]], List]);
  AA = Transpose[Map[Coefficient[ExtNL, #] &, Xvect]] /.
    Inner[Rule, Join[Xvect, Contrl], ConstantArray[0, Length[Join[Xvect, Contrl]]],
  BB = Transpose[Map[Coefficient[ExtNL, #] &, Contrl]] /.
    Inner[Rule, Join[Xvect, Contrl], ConstantArray[0, Length[Join[Xvect, Contrl]]],
  {K1, P, Eigs} = LQR[AA, BB, QQ, RR] ;
  V0 = (Xvect.P.Transpose[{Xvect}]) // Expand;
                 1
  LinCtrl = - --- * Inverse[RR].Transpose[BB].Jacob[V0[[1]], Xvect];
                 2
  Print[
   " The linear controller associated to the first order linear approximation is :
  Print[FirstOrderController, "=", LinCtrl // FullSimplify];
  (* Parametrization of the first and second order control law *)
  Ctrl1 = c1 * y1^3 + c2 * y2^3 + c3 * y3^3 + c4 * y4^3 + c11 * y1 * y2 * y3 +
    c22 * y1 * y3 * y4 + c33 * y2 * y3 * y4 + d11 * y1^2 * y2 + d12 * y1^2 * y3 +
    d13 * y1^2 * y4 + d22 * y2^2 * y1 + d23 * y2^2 * y3 + d24 * y2^2 * y4 + d33 * y3^2 * y1 +
    d32 * y3^2 * y2 + d34 * y3^2 * y4 + d44 * y4^2 * y1 + d42 * y4^2 * y2 + d43 * y4^2 * y3;
  Ctrl2 =  h1 * y1^4 + h2 * y2^4 + h3 * y3^4 + h4 * y4^4 + h11 * y1 * y2 * y3 * y4 +
    h22 * y1^2 * y2^2 + h21 * y1^2 * y3^2 + h23 * y1^2 * y4^2 + h33 * y2^2 * y3^2 +
    h31 * y2^2 * y4^2 + g11 * y1^3 * y2 + g12 * y1^3 * y3 + g13 * y1^3 * y4 +
    g22 * y2^3 * y1 + g23 * y2^3 * y3 + g24 * y2^3 * y4 + g33 * y3^3 * y1 + g32 * y3^3 * y2 +
    g34 * y3^3 * y4 + g44 * y4^3 * y1 + g42 * y4^3 * y2 + g43 * y4^3 * y3;
  VectorList = {y1^3, y2^3, y3^3, y4^3, y1 * y2 * y4, y1 * y3 * y4, y1 * y2 * y3,
    y1^2 * y2, y1^2 * y3, y1^2 * y4, y2^2 * y1, y2^2 * y3, y2^2 * y4,
    y3^2 * y1, y3^2 * y2, y3^2 * y4, y4^2 * y1, y4^2 * y2, y4^2 * y3};
  VectorList2 = {y1^4, y2^4, y3^4, y4^4, y1 * y2 * y3 * y4, y1^2 * y2^2, y1^2 * y3^2,
    y1^2 * y4^2, y2^2 * y3^2, y2^2 * y4^2, y1^3 * y2, y1^3 * y3, y1^3 * y4, y2^3 * y1,
    y2^3 * y3, y2^3 * y4, y3^3 * y1, y3^3 * y2, y3^3 * y4, y4^3 * y1, y4^3 * y2, y4^3 * y3}
  VectCoef1 = {c1, c2, c3, c4, c11, c22, c33, d11, d12, d13, d22,
```

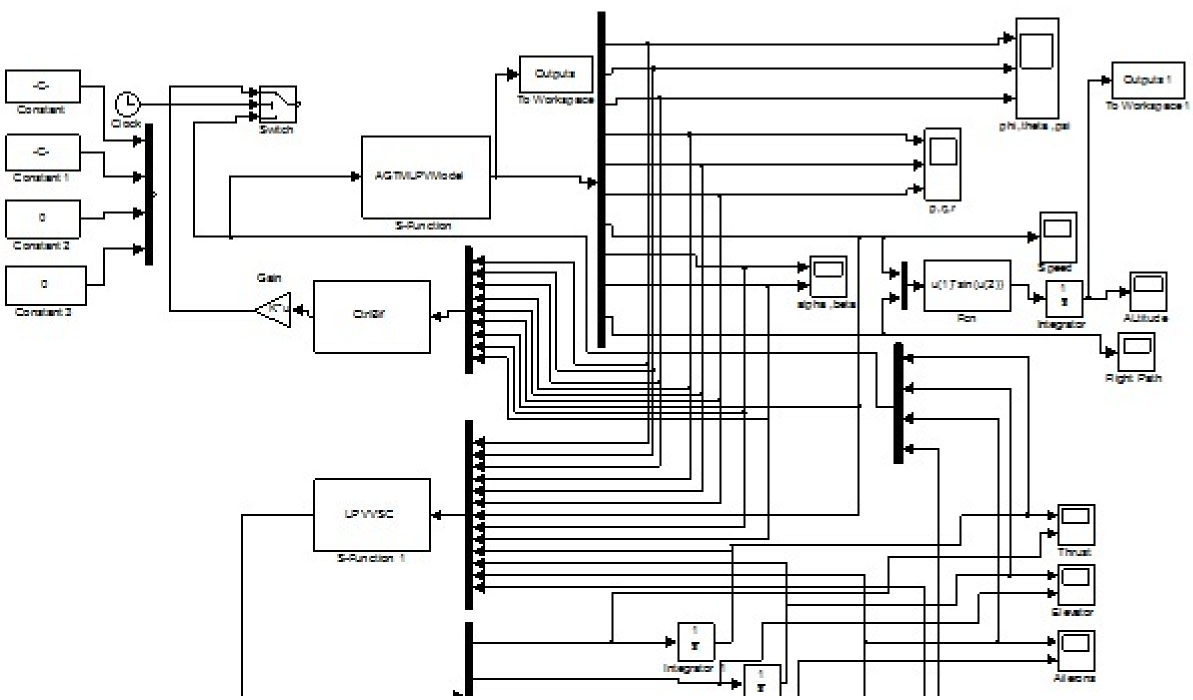


Figure 7.2: Implementation of the Recovery Procedure In MatLab Simulink

LPV and DAE for Multiple Models Algorithms

```
For[i = -m1, i ≤ m1, For[j = -n1, j ≤ n1,
   {AAd = (AA /. {s1 → i * h1, s2 → j * h2});
    BBd = (BB /. {s1 → i * h1, s2 → j * h2});
    Print[AAd // MatrixForm];
    Print[BBd // MatrixForm];
    UU = BBd.RR.Transpose[BBd];
    {KKn, PPn, Eign} = LQR[AAd, BBd, QQ, RR];
    Print[KKn // MatrixForm],
    Print[PPn // MatrixForm],
    Print[Eign] (* Closed loop eigenvalues *),
    Re[Take[Eign, 2]],
    Im[Take[Eign, 2]],
    Pts = Table[{Re[Print[Eign][[k]]], Im[Print[Eign][[k]]]} , {k, 1, 9}],
    (* Reals and Imaginary Part of the closed loop eigenvalues *)
    Print[Eigenvalues[AAd] // MatrixForm], (* Open loop eigenvalues *)
    ListPlot[Pts, PlotStyle → {PointSize[0.03], RGBColor[0, 0, 1]}, PlotRange → All],
    {UUn, WWn, VVn} = SingularValues[AAd],
    Print[WWn // MatrixForm],
    Mm = Max[WWn],
    Mn = Min[WWn],
    Ratio = Mn / Mm,
    Print[Mm, Mn, Ratio],
    Print[ControllablePair[AAd, BBd]]
      (* Test of controllability while varying the parameters in the design *)
     ObservablePair[AA, CC] },
   j++]
  i++]

MyDaeDisc2[f_List, g_List, x_List, y_List, x0_List, y0_List, u_, n1_Integer,
   n2_Integer, h_] := Module[{z, T, xI, yI, z1, f0, H, F, F1, z2, gg, GG, s},
   xI = x0;
   yI = y0;
   gg = g /. Inner[Rule, x, xI, List];
   GG = Jacob[gg, y];
   T = MyNewton2[gg, GG, y, yI, n1];
   f0 = f /. Inner[Rule, x, xI, List];
   H = f0 /. Inner[Rule, y, T, List];
   F = Join[((x - xI) - (1 / 2) * h * (H + f)), (g)];
   z = Join[x, y];
   F1 = Jacob[F, z];
   z1 = Join[xI, yI];
   z2 = MyNewton2[F, F1, z, z1, n2];
   xI = Take[z2, Length[x]];
   yI = Take[z2, -Length[y]];
   {xI, yI}
  ];
```

Bibliography

[1] Foster, J. V.;Cunningham K.;Fremaux C. M.;Shah G. H.;Stewart E. C.;Rivers A. C.; Wilborn, J. E.;Gato,W.*Dynamic Modeling and Simulation of Large Transport Airplane in Upset Conditions.* AIAA, GNC, 15-18 August 2005

[2] Krahe,C. Airbus *fly-by-wire at a glance.* Fast Airbus Technical Digest,20,Toulouse, December 1996

[3] NASA: *Aerodynamic Modeling of Large Transports for Upset Conditions.* Stage 2 EURS Progress Report for NAS1-00106 Task 1010, October 31, 2003.

[4] FAA, *FAA Aerospace Forecast Fiscal Years 2006 - 2017.* Federal Aviation Administration 2005.

[5] *www. CAA.CO.UK*

[6] NRC, *Aviation Safety and Pilot Control* - *Understanding and Preventing Unfavorable Pilot-Vehicle Interactions.* National Academy Press, Washington, DC 1997.

[7] Holloway C. M.,Johnson C. W. *How Past Loss-Of-Control Accidents may Inform Safety Cases For Advanced Control Systems on Commercial Aircraft.* Preceedings of the IET 1st Int. Conf. on System Safety, 6-8 June,2007

[8] NTSB, *Loss of Pitch Control During Takeoff AirMidwest Flight 5481.* NTSB/AAR-04/01, February 26 2004

[9] FAA,Upset Recovery Industry Team. *Airplane Upset Recovery Training Aid.* August 6, 2004.

[10] JianLiang Wang; N. Sundararajan. *Extended Nonlinear Flight Controllers Design for aircraft* Automatica, Vol. 32, No 8, PP. 1137-1193 1196

[11] William Garrard, John M. Jordan, *Design of Nonlinear Flight Automatic Control.* Automatica, Vol. 13, pp. 497-505, 1977

[12] Nerandra K.S.; Balakrishnan. *Adaptive control using multiple models.* IEEE Trans. Aut. Cont.Vol. 42, No 2, PP. 171-187, 1997

[13] P. S. Maybeck, *Multiple Models Adaptive algorithms for detecting and Compensating Sensors and Actuators/Surfaces Failures in Aircraft Flight Control Systems.* Int. Journal of Robust Control, Vol. 9, No. 14, pp. 1051 - 1070, 1999

[14] Bodson M. Groszkiewicz J. *Multivariable Adaptive Algorithm for reconfigurable Flight Control.* IEEE Trans. On Control System. Vol. 5, No 2 , PP 217-229, 1997

[15] G. Tao; Joshi, S. M.; Ma X. *Adaptive State Feedback and Tracking control of Systems with actuators failures.* IEEE Trans on Aut. Cont. Vol. 46. No 1, PP. 78-95 2001

[16] S. Thomas, H. G. Kwatny and B. C. Chang, *Nonlinear Reconfiguration of Asymmetric failures for a Six Degree-of-Freedom F-16.* Proceedings American Control Conference 2004, June 2004, pp 1823-1828

[17] S. Thomas, H. G. Kwatny and B. C. Chang, *Bifurcation Analysis of Flight Control Systems.* Proceedings International Federation of Automatic Control Triennial World Congress, Prague, July 2005.

[18] Robert Stengel, W. B. Nixon *Stalling Characteristics of a general Aviation Aircraft.* Journal of Aircraft, Vol. 19, No. 6, June, 1982

[19] Jon Roskam *Airplane Flight Dynamics and Automatic Flight Controls.* Roskam Aviation and Engineering Corporation, Library of Congress, 1979

[20] Vidyasagar, M. , C. A. Desoer *Feedback Systems: Input-Output properties* Academic Press, New York, 1975

[21] S. Thomas, *Reconfigurable and Bifurcation in Flight Controls*,Drexel University,2005

[22] B-C Chang, Gaurav Bajpai, Harry G. Kwatny *A Regulator Design to Address Actuator Failures.* Proceedings of the 40th IEEE Conference on Decision and Control, Vol. 2, 2001

[23] H. G. Kwatny, W. H. Bennett and J. Berg, *Regulation of Relaxed Static Stability Aircraft.* IEEE Trans. Auto. Contr., Vol. 36, No. 11, November 1991, pp. 1315-1323.

[24] Feld'Baum A. A. *Dual Control Theory I - IV* Automation and Remote Control.Vol 21,PP.874-1033, 1960

[25] John Legyros; Data N. Gogbole; Shankar Sastry. *Multiagent Hybrid System Design using Game Theory and Optimal Control.* Int. Machines and Robotic Lab. Univ. Cal.Berkeley,CA 94720

[26] Amitab, S., Girish D., Debasish G. *Sysnthesis of Nonlinear Controller to Recover an Unstable Aircraft From Postall Regime.* J. Guidance, Control and Dynamics. Vol. 22, No. 5 , September-October 1999.

[27] Lemmon. M.D., Markosky,I. *A tutorial introduction to supervisory hybrid systems.* Technical report of the ISIS group, University Notre Dame. 1998

[28] Peter A. Cummings.*Continuation Methods for Qualitative Analysis of Aircraft Dynamics.* NASA/CR-NIA Report No. 2004-06

[29] E. A. Morelli.*Global nonlinear Aerodynamic modeling using Multivariate Orthogonal Functions,* journal of aircraft,vol. 32, Issue2, 1995, pp. 270-277, 1995

[30] Suba Thomas, Gaurav Bajpai, Harry G. Kwatny, Bor-Chin Chang, *Nonlinear Dynamics, Stability & Bifurcation in Aircraft: Simulation and Analysis Tools.* Proceedings of 2005 AIAA Guidance, Navigation, and Control Conference, August 15-18, 2005.

[31] R. Seydel; *Practical Bifurcation and stability Analysis.* Springer Verlag. New York; 1994

[32] N.Ananthkrishnan, Nandan K. Sinha;*Level flight trim stability Analysis Using Extended Bifurcation and Continuation Procedure.* Journal of Guidance,Vol.24, No. 6, pp.1225-1228,1997

[33] M. G. Goman, G. I. Zagainov, A. V. Khramtsovski *Application of Bifurcation Methods to Nonlinear Flight Dynamics Problems.* Progress in Aerospace, vol.33, pp. 493-520, 1997

[34] Pierre - Etienne Aubin *Bifurcation Analysis and Control of a Fighter Aircraft.* Massachusetts Institute of Technology, 1993

[35] R. K. Merha, W.C. Kessel, J. V. Carroll;*Global Stability and Control Analysis of Aircraft at High Angles of Attack.* Report ONR-CR215-248-1, 1977

[36] D.Nesie *Analysis of Minimum Phase Properties for Non-Affine Nonlinear Systems.*IEEE,CDC, San Diego, California USA, December 1997

[37] Etkin B., L. D. Reid *Dynamics of Flight, Stability and Control.* John Wiley and Sons, New York, 1996

[38] Yiguang Hong.*Finite Time Stabilization and Stabilizability of a class of Controllable Systems.* Systems & Controls Letters. Vol 46, pp. 231-236, 2002

[39] C.W. Wampler, A. P. Morgan, A.J. Sommese. *Numerical Continuation Methods for solving Polynomial Systems Arising in Kinematics.* Journal of Mechanical Design, Vol.112, pp. 59-68,1990

[40] Jean-Etienne T. Dongmo, Harry G. Kwatny. *Nonlinear Robust Control System Design with Quantifier Elimination Using Mathematica.* Master thesis,Drexel University,2006

[41] S. Unnikrishnan and J. V. R. Prasad *Carefree Handling using Reactionary Envelope Protection Method.* in AIAA Guidance, Navigation, and Control Conference Keystone, Colorado, 2006.

[42] K. H. Well, *Aircraft Control Laws for Envelope Protection.* in AIAA Guidance, Navigation and Control Conference Keystone, Colorado, 2006.

[43] Claire Tomlin, John Legyros, Shankar Sastry *Aerodynamic Envelope Protection Using Hybrid Control.* Proceedings of the ACC. Philadelphia, pp. 1793-1796,1998

[44] John Lygeros *On Reachability and Minimum Cost Optimal.* Automatica, Vol. 40, pp. 917-927, 2004

[45] Sethian J. A. *Level Set Methods and Fast Marching Methods.* Cambridge Univ. Press. New York, pp. 305-308, 1999

[46] Goman, M. G.; A. V. Khramtsovsky, et al.*Aircraft Spin Prevention/Recovery Control System.* TSAGI.1993

[47] John Legyros;Claire Tomlin;Shankar Sastry. *Multi-objective Hybrid Controller Synthesis: Least Restrictive Controls.* Proc.CDC,San. Diego.1997

[48] Jean-Etienne T. Dongmo, Harry G. Kwatny, B-C Chang, C. Belcastro, Gaurav Baijpai, Murat Yasar. *Aircraft Accident Prevention and recovery: Loss-of-Control Analysis.*,GNC, AIAA,Chicago 2009

[49] Karl. J. Astrom *Interactions between Excitation and unmodeled Dynamics in Adaptive Control.* Proc. Of the CDC, Las Vegas, December, 1984.

[50] Bandu N. Pamadi. *Performance, Stability, Dynamics and Control of Airplane.* AIAA, Hampton, VA, 2003

[51] F. Stengel, *Flight Dynamics.* Princeton and Oxford Review, 2004

[52] Robert F. Stengel, Paul W. Berry *Stability and Control of Maneuvering High Performance Aircraft.* NASA, Washington, D.C, April 1977

[53] Richard Schevell. *Fundamentals of flight.* Princeton New-Jeysey,1980

[54] Eric F. Charlton. *Numerical Stability and Control Analysis towards Falling -Leaf Prediction Capabilities of Splitflow for two Generic High Performance Aircraft Models.* NASA/CR-1998-208730

[55] Fred E. Weick *American Views on Spin Phenomena. Three Papers Read before the Eighteenth National Aeronautic Meeting of the Society of Automotive Engineers.* Aircraft Engineering, Vol. 11, pp. 282-284, 1930

[56] L. Fridman, A. Levant *High Order Sliding Mode Control in Engineering.* Slidng Mode Contol in Engineering; New York, 2002

[57] Eyad H. Abed, Hsien-Chiarn Lee *Nonlinear Stabilization of High Angle-of-Attack Flight Dynamics Using Bifurcation Control*, *IEEE*,Vol. 12, pp 1823-1828, 1996

[58] E.B. Lee; L. Markus *Foundations of Optimal Control Theory. John wiley*, New-York 1967

[59] Brockett,R. W. *Feedback Invariant for Nonlinear Systems.* IFAC Congress, Vol. 6, pp. 1115-1120, 1978

[60] Isidori, A. *Nonlinear Control Systems* 3rd Edition, Springer-Verlag London Inc,London,1995

[61] Jakubczyk B., Respondek, W. *On the Linearization of Control Systems.* Bull Acad Polonaise Set Ser. Sci. Math., Vol. 28,pp. 517-522, 1990

[62] Van der Shaft AJ, Neijmeir, H.*Nonlinear Dynamical Control Systems.* Springer-Verlag New York Inc., New York 1990

[63] Vidyasagar, M. *Nonlinear Systems Analysis.* Second Edition, Prentice-Hall Inc, NJ, USA, 1993

[64] Dayawansa W. P., Boothby, W. M, Elliot, D. *Global State and Feedback equivalence of Nonlinear Systems.* Systems and Control Letters, Vol.6, pp. 229-234,1985

[65] Xinhua W, Geng Yang, Wenli Xu, *Output Tracking for Nonlinear Nonminimum Phase Systems and Application to PVTOL aircraft.* International Journal of Systems Sciences. Vol. 39; pp.29-42,2008

[66] A. Levant, J.Z. Ben-Asher, R. Gitizadeh, I. Yaesh. *2-Sliding Mode Implementation in Aircraft Pitch Control.*, Automatica, 2002.

[67] R. J. Patton, P. M.Frank, R. N. Clark *Fault Diagnosis in Dynamic Systems: theory and Applications.*, Prentice Hall, New York

[68] Harry G. Kwatny, Jordan Berg *An Upper Bound on Structurally Stable Linear Regulation of a Parameter-Dependent Family of Control System.* Systems & Control Letters, Vol. 23, pp. 85 - 95, 1993

[69] S. K. Spurgeon *Choice of Discontinuous Control Components for Robust sliding mode perfromance.* International Journal of Control,Vol.53, pp. 163-179, 1991

[70] Laghrouche, *MIMO Practical High Order Sliding Mode Control*, IEEE, 2005

[71] Harry. G. Kwatny; B - C Chang.*Construction of Linear Families from Parameter-Dependent Nonlinear Dynamics.* IEEE Trans. On Aut. Ctrl. Vol. 43, No. 8, pp. 1143 - 1147, 1998

[72] Harry. G. Kwatny; H. Kim; *Variable Structure Regulation of Partial Linearizable Dynamic.* Systems and Controls Letters, Vol. 15; pp. 67-80, 1990

[73] Utkin, V. I. *Variable Structure Systems with Sliding modes: a Survey.* IEEE Transactions on automatic Control,Vol. 22, pp. 212 - 222, 1977

[74] Nguyen, L. T., Ogburn, M. E., Gilbert, W. P., Kibler,K.S., Brown, P. W.; Deal, P. L.*Simulator Study of Stall/PostStall Characteristics of a Fighter Airplane with Relaxed Longitudinal Static Stability.* NASA,T.P-1538,1979

[75] V. I. Utkin; *Sliding Mode in Control Optimization.* Berlin: Springer, 1992

[76] S. V. Emel'yanov, S. V. Koroin, L. V. Levantovsky *High Order Sliding Modes in control system.* Diff. Equ. 29(11),pp. 1627-1647, 1998

[77] Siraz-Ramirez,H; *On the Sliding Mode Control of Nonlinear Systems.* Systems and Control Letters, Vol. 19,pp. 303 - 312, 1992

[78] Jean-Etienne T. Dongmo, Harry G. Kwatny, Chika Nkwampa, Gaurav Bajpai, Carole Teolis, *Variable Structure Control of a Fault Tolerant Control System for Induction Motors.*, IEEE, ASNE, 1997

[79] Young K. D., Kokotovic, P. V., *Actuator and Sensor Dynamics in Multivariable Feedback Systems.* Proc. CDC., Dec. 1978

[80] A. Levant, A. Pridor, J. Z. Ben-Asher, R. Gitizadeh, I. Yaesh. *2-Sliding Mode Implementation in Aircraft Pitch Control* International Journal of Control. 1993

[81] Daniel,C.Lluch. *Analysis of the Out-of-Control Falling Leaf Motion using a Rotational Axis Coordinate System.* Master Thesis, Virginia Polytechnic Institute and State University, 2009

[82] Gaurav Bajpai *Reconfigurable Control of Aircraft Undergoing Sensor and Actuator Failure.* PhD. Thesis, Drexel, University, Philadelphia, December, 2002.

[83] Robert T. N. Chen; Fred D. Newell; Armo E. Shelhorn; *Development and Evaluation of An Automatic Departure Prevention System and Stall Inhibitor for Fighter Aircraft.* Calspan Corporation Buffalo, New York, USA, 1979

[84] M. K. Khan, S. K. Spurgeon, A. Levant *Simple Output-Feedback 2-Sliding Controller for Systems of relative degree two.* IEEE, 2009

[85] S. Bhat; D. Bersntein; *Geometric Homogeneity with applications to finite time stability.* Math. Control. Signals Systems. Vol. 17, pp. 101-127, 2005

[86] Kothare, V. M., Campo J. P; M. Morari, Nett, N. C. *A Unified Framework for the study of Antiwindup Designs.* Automatica, Vol. 30, No 12, pp. 1869 - 1883, 1994

[87] Yamada, K; Funami Y. A. *Design Method of Robust Servo Internal Model Control with control Input Saturations.* Trans. Jpn Soc. Mech. Eng. Vol. 67, No. 664, pp. 3856-3862, 1998

[88] Marcopoli,R.V; Phillips, M. S; *Analysis and Synthesis Tools for a Class of Actuators-Limited Multivariable Control Systems: Linear Matrix Inequality Approach.* Int. Journal of Robust et Nonlinear Control. Vol.6; pp. 1045-1053, 1996

[89] Fen Wu; Grigoriadis, M. K. *Actuator Saturation Control via Linear Parameter Varying Control Methods, Actuators Saturation Control.*, Marcel Decker, Inc; pp. 273-298,2002

[90] Noriaki Itagaki; H. Nishimura; K. Takagi.*Design Method of Gain scheduled Control System Considering Actuator Saturation.*,JSME International journal,Vol. 46; No. 3,pp.953-959,2003

[91] Suba Thomas; Harry G. Kwatny; B-C Chang; *Lie Transform Construction of Exponential Normal Form Observers.* 4th Symposium on Robust Control Design,Vol. WM01-2, Milano,Italy, June 2003

[92] Rynaski, E. G. *Flight Control Synthesis Using Robust Output Observers.* Guidance, Navigation and Control, pp. 825 - 831,1982.

[93] Christine M. Belcastro, Celeste M. Belcastro *Application of Failure Detection, Identification and Accomodation Methods for Improved Aircraft Safety.* Proceedings of the ACC, Airlington, VA, June 25 - 27, 2001

[94] Austin M. Murch.*A Flight Control System Architecture for The NASA AirSTAR Flight Test Infrastructure.* AIAA, Guidance,Navigation and Control Conference, August 18-21, Hawai, 2008

[95] Harry G.Kwatny, Gilmer L. Blankenship *Nonlinear Control and Analytical Mechanics: A Computational Approach.* Birkhauser, Boston, 2000

[96] A. Levant *Sliding order and Sliding accuracy in Sliding Mode Control.*, International Journal of Control, Vol. 58, No. 6, pp. 1247 - 1263, 1993

[97] J. Descusse, C.H. Moog, *Decoupling with Dynamic Compensation for Strong Invertible Affine Nonlinear Systems.* International Journal of Control, Vol. 42, No. 6, pp. 1387 - 1398, 1985

[98] Gary K. Hellmann; Robert B. Crombie, *High Angle of Attack, Flying qualities and Departures Criteria development.* Report, Airforce Lab, 1979

[99] Luat T. Nguyen, John V. Foster *Development of a Preliminary High Angle of Attack Nose Down Pitch Control requirement for High Performance Aircraft.* NASA Langley Laboratory, 1990

[100] D. Nesie, E. Skafidas, I. M.Y. Mareels, J. Evans *Minimum phase properties for inputs Nonaffine Nonlinear Systems.* IEEE on Aut. Control, Vol. 44, 1999

[101] Robert Weismann *Preliminary Criteria for predicting Departure Characteristics Stall/Spin susceptibility of fighter-Type Aircraft.* Journal of Aircraft, vol. 10, No. 4, 1990

[102] Jurgen Ackermann, *Robust Control design using multiples models.* IEEE, Trans. on Automation Control. 1980

[103] Daniel Liberzon. *Switching in Systems and Control.* Birkhauser, 2003

[104] Michael Branicky *Stability of Switched and Hybrid Systems.* Conference on Decision and Control, December, 1994

[105] Michael Athans *Foundations of Optimal Control.*, John & Wiley, New York, 1966

[106] Claire Tomlin, John Lygeros, Shankar Sastry *Aerodynamic Envelope Protection Using Hybrid Control*,ACC, June, 1998

[107] Harry G Kwatny, Gaurav Bajpai, Adam Beytin, Suba thomas, M. Yasar *Nonlinear Modeling and Analysis Software for Control Upset Prevention and Recovery of Aircraft.* AIAA, 2008

[108] M. G. Goman, A. V. Kramtsovski *Application of Continuation and Bifurcation Methods to the design of Control System*, Phil. Trans. R. Soc. Lond. pp. 2277 - 2295, 1998

[109] Annakhrishhnan, A, G*Aircraft Spin Recovery with and without Thrust Vectoring Using Nonlinear Dynamic Inversion.* Journal of Aircraft Vol. 42, No. 6, 2005

[110] Liebst S. Brad *The Dynamic, Prediction, and Control of Wing Rock in High Performance Aircraft.* The Royal Society, Vol. 356 No. 1476, 1998

[111] Austin M. Murch, J. V.Foster *Recent NASA on Aerodynamic Modeling of Post Stall and Spin Dynamics of Large Transport Airplane.* NASA Langley Research Center, Hampton, VA, 2008

Johnston:1980 Donald E. Johnston, David G. Mitchell, *Investigation of High Angle - Of - Attack Maneuver - Limiting Factors: Part II, Pilot Assessment of Birhle Departure Criteria.* Airforce Reports Lab. 1980.

[112] M. G. Goman, A. V. Khramtsovsky *Computational Framework for Investigation of Aircraft Nonlinear Dynamics.* Advanced in Engineering Software Vol.39, pp. 167 - 177, 2008

[113] EN. Abdulwab C. Hongquan *Characterization of States Variables in Post - Stall Dynamic of High Performance Aircraft.* Journal of Aerospace Engineering, Vol. 222,Part G. 2007.

[114] Gary K. Hellmann, Robert B. Crombie *High Angle - Of - Attack Flying Qualities and Departure criteria Development.* Airforce Dynamic Lab.,1980

[115] Jay M. Brandon *Dynamic Stall Effects and Application to High Performance Aircraft.* NASA Langley Research Center Hampton, Virginia, report 23665.

[116] Airforce Flight Center *Approach - to - Stall/Stall/Post - Stall Evaluation*, AFFTC - TR - 75 - 3. Edwards AFB,CA: HQ AFFTC.

[117] Christopher J.Bett, Michael D. Lemmon *Bounds Amplitude Performance of switched LPV Ssystems with Applications to Hybrid Systems.* Automatica 35, pp. 491 - 503, 1999

[118] Michael Lemmon, Christopher Bett *Safe Implementation of Supervisory Commands*, Department of Electrical Engineering Notre Dame, 2000.

[119] Mani. M.Toussi, Idin Karuei, Shahin Hastrudi-zad, Amir G. Aghdam *Supervisory Control of Switching Control Systems.* Systems and Control letters, Vol. 57, pp. 132 - 141, 2008

VITAE

Education

Doctor of Philosophy, Mechanical Engineering & Mechanics, Drexel University, June 2010

Master of Science and Engineering, Mechanical Engineering & Mechanics, Drexel University, June 2006

Bachelor of Science, Mechanical Engineering & Mechanics, Drexel University, June 2006

Bachelor of Science, Applied Mathematics, University of Yaounde I, September, 1999

Research Interests

Flight Control, Hybrid Systems, Switching Systems, Ground and Space Vehicles, Robotics & Automation.

Research Experience

Graduate Assistant, September 2005 - June 2010
Systems and Controls Labs, Mechanical Engineering & Mechanics Department, Drexel University, PA, USA

Teaching Experience

Teacher Assistant, September 2005 - June 2010
Mechanical Engineering and Mechanics Department, Drexel University, Philadelphia, PA, USA

Publications

1. Harry G. Kwatny, Jean-Etienne T. Dongmo, Robert Allen, B - C Chang, Gaurav Bajpai. *Loss - Of - Control: Perspective on Flight Dynamics and Control of Impaired Aircraft.* AIAA, Guidance,Navigation and Control. Toronto 2010. Submitted.

2. Jean-Etienne T. Dongmo, and Harry.G. Kwatny. *Aircraft Loss - Of - Control Recovery Using High Order Sliding Control.* AIAA, Guidance, Navigation and Control. Toronto 2010.Submitted, Rejected, to be revised.

3. Jean - Etienne T. Dongmo, Harry G. Kwatny. *Aircraft Loss - Of - Control Recovery Using Nonlinear Smooth Regulators.* AIAA, Journal of Guidance, Dynamics and Control, To be submitted, 2010.

4. Jean - Etienne T. Dongmo, Harry G. Kwatny, Christine Belcastro, Gaurav Bajpai, Murat Yasar *Aircraft Accident Prevention and Recovery: Loss - Of - Control Analysis.* AIAA, Guidance, Navigation and Control, Chicago, 2009.